



# A Bayesian study of changes in volatility of Bitcoin

## *Un estudio Bayesiano de los cambios de volatilidad del Bitcoin*

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### **Abstract**

This paper is aimed at studying a MS-GARCH model applied to Bitcoin. The Bayesian estimation of the model shows that Bitcoin's volatility can be modelled using two states of volatility, high and low. The modelled volatility is not stable over time. Twenty eight periods of high volatility were found, the largest period of volatility occurred during 2013. The findings help explain what happened during these high volatility periods.

*JEL code:* C11, C22, G17

*Keywords:* Bitcoin; volatility; MS-GARCH; Bayesian estimation

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## Resumen

Este artículo está enfocado en estudiar un modelo MS-GARCH aplicado al Bitcoin. La estimación bayesiana del modelo muestra que la volatilidad del Bitcoin puede ser modelada utilizando dos estados de volatilidad, alto y bajo. La volatilidad modelada no es estable en el tiempo. Se encontraron veintiocho periodos de alta volatilidad, el periodo de volatilidad más grande ocurrió durante el 2013. Los hallazgos ayudan a explicar qué pasó durante estos periodos de alta volatilidad.

*Código JEL:* C11, C22, G17

*Palabras clave:* Bitcoin; volatilidad; MS-GARCH; Estimación bayesiana

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## Introduction

The time series behaviour of Bitcoin's price has received a lot of attention lately. There is still a debate on the proper definition of its nature and to whether it is a currency, a commodity or something in between (Dyhrberg, 2016; Yermack, 2015). In order to be considered a currency, it should fulfil the main standard properties of being a unit of account (there are no financial statements valued in Bitcoins), a medium of exchange (it is accepted mainly for some online purchases) and it might be difficult to believe that, given the great swings in price of Bitcoin, anyone can consider Bitcoin as a suitable option to store value (Dwyer, 2015).

However, even if there is an ongoing debate around Bitcoin's nature, it could fit as an ideal asset for speculative purposes, since there is no underlying asset to relate its value to, and there are many open platforms to operate round the clock, allowing even for arbitrage opportunities (Makarov & Schoar, 2020). Baek & Elbeck (2015) report evidence on Bitcoin's internal volatility and the high volatile market of it. Kristoufek (2013) mentions that Bitcoin's behaviour is not influenced by standard economic or financial theories, since it is a digital currency, which is used for any businesses as an electronic payment medium and could be considered as a volatile and speculative asset (Cheung, Roca, & Su, 2015; Kristoufek, 2015; Pak Nian & Lee Kuo Chuen, 2015).

Even if Bitcoin is not related to fundamental economic variables, it is important to model and study the volatility of such a virtual currency, one of the main reasons being that, besides speculative purposes, Bitcoin -amongst other cryptocurrencies- has been gaining a prominent role as a mean of almost instantly -around an hour, corresponding to 6 confirmations from miners of about ten minutes each- transfer international currency, compared to traditional

international currency transactions that could take up to a week and depend on multiple exchange rates (Angel & McCabe, 2015; Antonopoulos, 2015; Li & Wang, 2017; Pieters & Vivanco, 2017). Another reason is that Bitcoin has proven certain hedging capabilities and advantages as a medium of exchange, similar to gold and the dollar (Dyhrberg, 2016). Bitcoin has also been shown to be a hedge against global uncertainty in the bull regime and at short investment periods (Bouri, Gupta, Tiwari, & Roubaud, 2017).

Bitcoin's volatility has been modelled by several extensions of the family of Generalized Autoregressive Conditional Heteroskedasticity (GARCH), e.g. Kurihara & Fukushima, (2018) studied Bitcoin's returns applying three different volatility extensions: Exponential (EGARCH), Component (CGARCH) and Power (PGARCH), and they found a difference between short-term and long-term volatility, from which it can be inferred that Bitcoin's price movements should be studied using other more robust techniques. Applying an ARCH and GARCH model, Bouoiyour & Selmi (2016) concluded that Bitcoin's market is still under development. Bouri, Azzi, & Haubo Dyhrberg (2017), with the aid of a GARCH model, examined whether there was a yield-volatility relationship before and after Bitcoin's price crash in 2013; they found a significant inverse relationship between shocks and volatility before the crash and no significant relationship after the crash. Katsiampa, (2017) estimated Bitcoin's volatility using a GARCH battery of tests, including Threshold (TGARCH), EGARCH, Component with Multiple Threshold (CMT-GARCH), Asymmetric Component (AC-GARCH), Asymmetric Power ARCH (APARCH), and concluded that it was very important to have a conditional variance component, in order to properly model Bitcoin's volatility. In the same fashion, Baek & Elbeck, (2015), using Katsiampa's model, found that the understanding of Bitcoin's volatility could be a useful tool for portfolio and risk management. In this line, Bouri, Gil-Alana, Gupta, & Roubaud (2018) showed the persistence of Bitcoin's volatility in the long-term and short-term, while Bariviera (2017) found the same results applying the Hurst exponent. In contrast with the above, and based on (Bauwens, Dufays, & Rombouts, 2014), in the present document Bitcoin's returns and price volatility are modelled through GARCH processes with state dependent parameters over time, i.e., Markov switching parameters. Moreover, the model is selected through the Bayes Factor (BF) method. This methodology has not been previously applied to Bitcoin, and is advantageous because its parameters are selected exogenously.

The paper is organized as follows. Section 2 explains the methodology used to get the changes in volatility through Bayesian methods. Section 3 describes the data and results. Section 4 explains some events that occurred during the periods of high volatility. Finally, Section 5 concludes.

## Methods

The GARCH model introduced by (Bollerslev, 1986) is a model with fixed parameters, which implies that the persistence of the conditional variance is constant over time. A model that changes the persistence of such conditional volatility is given by a Markov process with discrete dynamics, such as the MS-GARCH model, which varies the parameters over time for a latent discrete Markovian process (Ardia, Bluteau, Boudt, Catania, & Trottier, 2019).

However, to discriminate between different GARCH models with or without state changes, Bauwens et al. (2014) suggested a Bayesian methodology that estimates the possible number of states or regimes using the Bayes Factor (BF) criterion, maximizing the value of the marginal likelihood. The simulation technique is based on Monte Carlo Particle Markov Chains (PMCMC), which combines a Sequential Monte Carlo (SMC) and Markov Chains with Monte Carlo Simulation (MCMC), see (Bauwens et al., 2014) for all details of the methodology.

Consider the MS-GARCH model given by

$$y_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \omega_{s_t} + \alpha_{s_t} y_{t-1}^2 + \beta_{s_t} \sigma_{t-1}^2$$

$$\epsilon_t \sim N(0,1)$$

where  $s_t$  is an integer random variable taking values  $\{1, \dots, K+1\}$ . Let  $Y_t = \{y_1, \dots, y_T\}'$  and,  $S_T = \{s_1, \dots, s_T\}'$ ,  $T$  being the sample size, and a parameter vector given by  $\theta = (\omega_1, \dots, \omega_{K+1}, \alpha_1, \dots, \alpha_{K+1}, \beta_1, \dots, \beta_{K+1})'$ . The latent state  $\{s_t\}$  is a first order Markovian process with recurrent transition matrix given by

$$P = \begin{pmatrix} p_{1,1} & \dots & p_{1,K} & 1 - \sum_{j=1}^K p_{1,j} \\ \vdots & \ddots & \vdots & \vdots \\ p_{K+1,1} & \dots & p_{K+1,K} & 1 - \sum_{j=1}^K p_{K+1,j} \end{pmatrix},$$

where  $p_{i,j} = \mathbb{P}[s_t = j \mid s_{t-1} = i]$ .

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The estimation of the model parameters,  $\theta$  and  $P$ , is done by marginal likelihood, a standard Bayesian criterion. However, such estimation is unfeasible for the sample size due to the path dependence problem, which occurs given the recursive nature of the GARCH process, since the conditional variance at time  $t$  depends on the entire sequence of regimes visited up to  $t-1$ . For

a great exposition on the subject of path-dependence in economics, see (David, 2007). This would require to integrate the likelihood function over  $(K+1)^T$  paths. In order to overcome such a problem, Bauwens et al. (2014) propose a Bayesian inference by using data augmentation, i.e., by treating  $S_T$  as a parameter, and sampling from the posterior distribution  $f(\theta, P, S_T | Y_T)$ , using a Gibbs sampling algorithm that iteratively draws from the following full conditional distributions: 1)  $p(S_T | \theta, P, Y_T)$ , 2)  $f(P | S_T, \theta, Y_T) = f(P | S_T)$  and 3)  $f(\theta | S_T, P, Y_T) = f(\theta | S_T, Y_T)$ . Based on an adaptive Metropolis-Hastings simulation, the proposed particle Gibbs algorithm generates a dependent sample of  $f(\theta, P, S_T | Y_T)$  and a full state vector is drawn from  $p(S_T | \theta, P, Y_T)$  with a conditional SMC algorithm.

## Data and Empirical Results

The data sample consists of daily prices,  $p_t$ , of Bitcoin per US dollar (B/US) from 12/19/2011 to 08/07/2017 for a total of 2,059 daily observation. The data was obtained from <http://data.bitcoinity.org>. The original data was transformed into series of continuously compounded percentage returns, in accordance with the following expression:  $r_t = 100 \cdot \ln(p_t - p_{t-1})$  where  $r_t$  is the return of the series of the B/US in period  $t$ ,  $p_t$  is the closing B/US price in period  $t$ , and  $p_{t-1}$  is the closing B/US price in period  $t-1$ . Fig. 1 shows the time series of  $p_t$  and  $r_t$ . The returns of B/US show that there might be different periods of volatility.

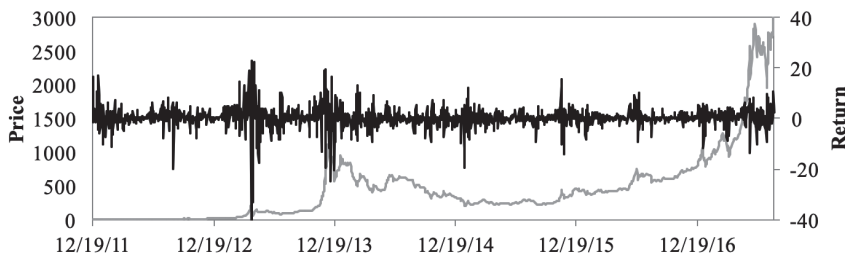


Figure 1. Prices and returns of B/US.

Table 1 (Panel A) shows the descriptive statistics of the returns of B/US. The return series is left-skewed, leptokurtic and non-normally distributed according to the Jarque-Bera statistic. Stationary of the returns series is tested using the Residual Augmented Least Squares (RALS) test proposed by (Im, Lee, & Tieslau, 2014) which does not require a specific density function

for the error term, see Table 1 (Panel B).

Table 1

## Descriptive statistics and unit root tests

Panel A	
Descriptive statistics of Bitcoin's returns	
Mean	0.33
Median	0.24
Minimum	-50.4
Maximum	22.58
Variance	15.53
Standard deviation	3.94
Skewness	-1.87
Kurtosis	31.51
Jarque-Bera statistic	70916.37*
Panel B	
Unit Root Test	
RALS statistic	-33.11**

Note: \* indicates that the null hypothesis of normality at 1% is rejected. \*\* indicates that the null hypothesis of stationary at 1% is rejected. Source: Authors' own.

Table 2 shows the Logarithmic Marginal Likelihood (LML) according to the Bridge Sampling technique (BS) and Logarithmic Bayes Factors (Log-BF). The best model is an MS-GARCH(2). In a Bayesian context, a more formal way to determine the appropriate number of states is to use the Log-BF, which indicates how likely one model is relative to another. The Log-BF is computed, when testing model A versus model B, *a posteriori* model A is more likely when  $\text{Log}(BF_{A/B}) > 1$ , assuming *a priori* model A is equally likely as model B (Voormanns, 2016). Higher values of the computed Log- BF correspond to stronger evidence in favour of model A. In this case, there is clear evidence that two states are indeed sufficient to model the volatility.

Table 2

Marginal log likelihood MS-GARCH and Log-BF

Regime	1	2	3
BS	-4889	<b>-4838.79</b>	-4857.72
Chib's	-4888.78	<b>-4835.46</b>	-4856.57
Log-BF BS	50.21	0	18.93
Log-BF Chib's	53.32	0	21.11

Note: The mean of the Marginal log likelihood simulation (BS technique).

Log-BF: log Bayes Factor with respect to the MS- GARCH(2). Source:

Authors' own.

The posterior means and standard deviation of the parameters, the transition probabilities, steady-state vector and duration of the MS-GARCH(2) model selected, are shown in Table 3. We find that the respective regime-specific sums for the MS-GARCH(2) model are given by 0.9349 and 0.8890, indicating covariance stationarity with high degrees of volatility persistence in both Markov-regimes (Reher & Wilfling, 2016) we establish a generalized two-regime Markov-switching GARCH model which enables us to specify complex (symmetric and asymmetric).

Table 3

Posterior means, transition probabilities, steady-state vector, and duration for MS-GARCH(2)

	1	2	Transition probabilities	Steady-state vector	Duration
$\sigma^2$	71.5043 (94.6411)	3.6991 (16.9797)	$\begin{pmatrix} 0.9921 & 0.0079 \\ 0.0073 & 0.9927 \end{pmatrix}$	$\begin{pmatrix} 0.4803 \\ 0.5197 \end{pmatrix}$	$\begin{pmatrix} 126 \\ 137 \end{pmatrix}$
$\omega$	4.6578 (0.3372)	0.4107 (0.1342)			
$\alpha$	0.6782 (0.0649)	0.3885 (0.0558)			
$\beta$	0.2567 (0.0564)	0.5605 (0.0904)			

Note: In columns two and three, the parameters at the posterior mean and standard deviations, in parentheses. In column four is the transition matrix. Column five presents the steady stationary vector. The last column is the state duration in days  $\frac{1}{1-p_{ij}}$ . The (local) unconditional variance  $\sigma^2$  is computed as  $\frac{\omega}{1-\alpha-\beta}$ . Source: Authors' own.

These results show that the transition probabilities are above 0.99, very close in both regimes, which indicates a high persistence. The probabilities of the steady state are of 48.03% for the high volatility, and of 51.97% for the low volatility, these are in 48% of the sample of the data in high volatility. The duration that remains in each state is 126 days and 137 days, respectively. Therefore, the high persistence of each state shows that once there is a change of state, the volatility of the returns remains on high or low conditional volatility for several periods of time.

Fig. 2 shows the transition of different changes and periods of high and low volatility from grey to white, respectively. A period of high volatility can be seen in 2013, which coincides with the change in exogenous volatility assumed by (Cheah & Fry, 2015; Urquhart, 2016).

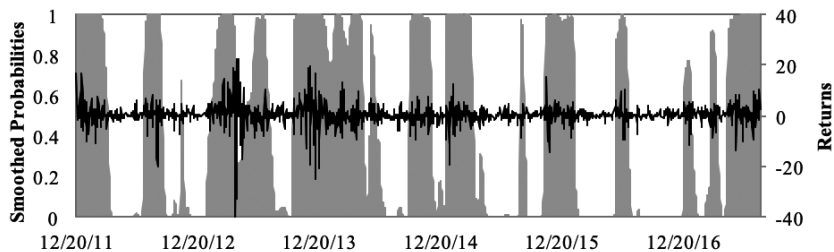


Figure 2. High and low volatility periods for returns of B/US.

Source: Authors' own

The results are consistent with (Katsiampa, 2017) who finds that Bitcoin volatility is not constant over time, and (Bouoiyour & Selmi, 2016) who found two periods of volatility, high and low volatility. However, with this methodology one can find the periods of high and low volatility and the selection of the MS-GARCH model according to the adjustment of the data, which is endogenously, through the marginal probability. Forecasting of periods of high and low volatility might also be done using the model discussed.

### Discussion of events of high volatility

In this section we present some key moments of high volatility that are captured with the model and some of the events that occurred during that period. Given that volatility is inherently related to risk in financial markets, it is of great importance to be able to capture high volatility with a model. In order to validate that the model has captured the periods of high volatility, in this section we relate such periods with the corresponding risky events.



Periods of high volatility: 12/20/11 - 03/22/12, 03/23/12 - 07/09/12, 07/10/12 - 09/03/12, 09/04/12 - 10/28/12, 10/29/12 - 11/01/12, 11/02/12 - 01/27/13

Events: The peak of the so-called great Bitcoin bubble occurred during 2011. There were price fluctuations that ranged from \$2 USD in November, 2011 to \$7.20 USD in early January, 2012, falling again and rising up to \$15.25 USD (Lee, 2013). The news of the investigation by the FBI on Bitcoin was known by August 2013 (Clinch, 2013). The Department of Homeland Security issued a letter to the Committee on Homeland Security and Governmental Affairs asking for information about Bitcoin's use by investors and entrepreneurs. However, their main concern was on the possible use of Bitcoin by criminals since it allowed them to "operate in the shadows". The *Bitconica* trading site was hacked on March 1<sup>st</sup>, 2011 and 43,000 Bitcoins were stolen. Even if the site announced to their users that it would respond for the Bitcoins hacked, and thus ensure that their funds were safe, a lot of volatility permeated into the market, see (Morse, 2017).

Periods of high volatility: 01/28/13 - 04/09/13, 04/10/13 - 05/11/13, 07/23/13 - 10/01/13, 10/02/13 - 05/01/14

Events: In the beginning of 2013, there was a Bitcoin boom. However, after doubling the 2012 price by early March, Bitcoin's price collapsed (\$34 USD) on the 6<sup>th</sup>, before rising again to \$45 USD. The same behaviour of ups and downs happened again by the middle of March, ending the month in a price of up to \$90 USD (Lee, 2013). In March 18, 2013, the Financial Crimes Enforcement Network issued a guide to regulate users, administrators and exchangers of virtual currencies like Bitcoin (FCEN, 2013). In such a document, there is a clear distinction between a "real" currency, i.e., the coin and paper money issued by a country that is designated as a legal tender, circulates and is accepted as a medium of exchange. For the effect of the aforementioned regulation, they also define "virtual" currency, which operates as a medium of exchange in some environments, and "convertible" virtual currency, which has an equivalent value in real currency and is the effect of such a regulation. Also in March, Cyprus accepted a bailout from the European Central Bank of 10 billion Euros (Matthews, 2013). Such a bailout was the result of a financial crisis of the country, which had its origins in a spillover from the Greek financial crisis (Mink & de Haan, 2013). The conditions of the bailout included a levy collected from private bank accounts with balances over \$100,000 Euros. In order to escape the levy, some investors bought other assets, like Bitcoin, which originated a spike in demand of the virtual currency, and a high volatility period. Furthermore, from May to August 2013, a lot of Bitcoin trading companies closed indefinitely due to American banks shutting down their accounts, arguing that operations with Bitcoin were

too risky. Amongst the companies that went out of business in the United States are: Mt. Gox, Bitfloor, BitInstant, Coinabul, Tradehill, Bitbox and Bitspend (Badev & Chen, 2015).

Periods of high volatility: 05/22/14 – 06/07/14, 06/08/14 – 09/16/14, 09/17/14 – 11/22/14, 11/23/14 – 01/06/15

Events: The price of Bitcoin had a sharp drop on February 6<sup>th</sup>, 2014, when Mt. Gox –a Bitcoin exchange based in Japan that handled over 70% of transaction worldwide from 2011 to 2014– experienced a trading stoppage due to a distributed denial-of-service (DDoS) cyber-attack (King, 2014). After such an attack, Mt. Gox filed for bankruptcy, after the announcement of the disappearance of Bitcoins worth almost \$500 million US dollars. A few months after, the price of Bitcoin went up again, by more than 50 times its price two years before. By October 7<sup>th</sup>, 2014, it was reported that 64,000 businesses were accepting payments in Bitcoin (Badev & Chen, 2015).

Periods of high volatility: 01/07/15 – 03/27/15, 03/28/15 – 08/15/15, 08/16/15 – 08/28/15, 08/29/15 – 10/28/15, 10/29/15 – 01/26/16

Events: Another popular Bitcoin exchange, Bitstamp, was hacked by January, 2015, stealing almost \$5 million US dollars in Bitcoins. The exchange shut down for eight days before opening again, assuring that no customer funds were stolen (Morse, 2017). 2015 was a year of many periods of high volatility for Bitcoin. Gold went down by 10 percent, U.S. stock indexes were almost flat and energy commodities fell more than 30 percent. However, Bitcoin went from \$313 dollars at the beginning of the year to \$175 in mid-January, jumping back up to \$430 at the end of the year (Rosenfeld, 2015).

Periods of high volatility: 01/27/16 – 05/30/16, 05/31/16 – 07/05/16, 07/07/16 – 12/20/16, 12/21/16 – 01/11/17, 01/12/17 – 03/10/17, 03/11/17 – 03/29/17, 03/30/17 – 04/28/17

Events: For the first time, Bitcoin price breached the \$1,000 mark (Das, 2017). In September 2017, China's regulators cracked down the market, by banning initial coin offerings (ICOs), and thus forcing some exchanges to end trading and moving out of the country. The price of Bitcoin had a price record of \$5,000 at the beginning of September, with a low of \$2,951 by the middle of the month, and finally recovering to \$4,204. Opposed to China, Japan showed its support to virtual currencies by recognizing crypto-currency exchanges under certain requirements, like strong computer systems and implementing tools to prevent fraud (Graham,

2017). Concerns about the manipulation of Bitcoin's boom started to appear (Rooney, 2018), and some argued in favour of coordinated price manipulation (Griffin & Shams, 2018).

## Conclusions

In this document, we studied dynamic changes in Bitcoin's price volatility. The changes found were classified as high and low volatility. The periods coincide with some of the documents that have studied the variance exogenously, which show that the parameters of the conditional variance are not constant over time. However, our contribution has been to choose the model that best fits the data through a Bayesian methodology, based in the methodology introduced by Bauwens et al. (2014).

State changes in conditional volatility may be due to the speed with which bitcoin reacts to the news (Bouoiyour & Selmi, 2016). The changes of state in the conditional volatility can be due to the rapidity with which it reacts to the news, there is no lag of days to process it, this may be due to the computer algorithm that avoids the possibility of arbitrage (Kristoufek, 2015). The volume of the market is another possible variable that could well cause changes in volatility. We leave for future research the study of factors that could determine the changes of state of said volatility, amongst which might be: market size, low liquidity, regulation, news events, shifting sentiment, inequality of wealth, and key role of speculation.

The modelling and understanding of volatility of financial assets, and in particular of cryptocurrencies, Bitcoin being the dominating one, is of great importance given the relationship that it holds with respect to risk, that is why we have focused our attention on the understanding of high volatility, and thus events of great financial risk. If Bitcoin is to be considered more as a currency of exchange and used for international transactions and as a hedge against other financial assets, the understanding of volatility's dynamics is vital.

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