Multiple technical interest rates: A contribution to strengthening the stability of pension system

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Abstract

In actuarial science relative to pensions and life annuities, it is a common assumption that the discount rate used to calculate the adequate reserve amount to cover future payments is equal to the expected long-term return rate of portfolios in which is invest. That assumption is inconvenient because it could lead fund managers take an excess risk in order to have greater profitability and ignore that each future cash flow should have a discount rate according with their pay date. This article demonstrates the existence of a suitable technical interest rate to discount each future payment, however these rates are not necessarily equal among them and the expected long-term return of the portfolio. In order to estimate those technical interest rates, it’s proposed to apply a risk model in each of the expected payments, which incorporates

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the fluctuations of the portfolio in which the actuarial reserves are inverted. Calculate adequate discount rates to determine actuarial reserves contributes to strengthening the stability of pension systems and the financial system in general.

*JEL code: G22, G23, G32*

*Keywords: Life annuity; Credit risk; Pension fund; Technical interest rate*

**Resumen**

En la ciencia actuarial de vida relativa a pensiones y rentas vitalicias es un supuesto usual que la tasa de descuento utilizada para calcular el monto de reserva adecuado para cubrir los pagos futuros, es igual a la tasa esperada de retorno de largo plazo de los portafolios en los cuales está invertida. Ese supuesto es inconveniente ya que podría llevar a los administradores de los recursos a tomar un exceso de riesgo con el fin de tener mayor rentabilidad e ignora que cada flujo de efectivo futuro debe tener una tasa de descuento acorde con su fecha de pago. En este artículo se demuestra la existencia de una tasa de interés técnico adecuada para descontar cada pago futuro y que dichas tasas no son necesariamente iguales entre sí, ni iguales al retorno esperado de largo plazo del portafolio. Para estimarlas se propone la utilización de un modelo de riesgo de incumplimiento en los pagos previstos, que incorpora las fluctuaciones propias del portafolio en el cual están invertidas las reservas actariales. Determinar las tasas de descuento adecuadas para calcular reservas actariales contribuye a fortalecer la estabilidad de los sistemas pensionales y del sistema financiero en general.

*Código JEL: G22, G23, G32*

*Palabras clave: Renta vitalicia; Riesgo de crédito; Fondo de pensión; Tasa de interés técnico*

**Introduction**

The calculation of the actuarial reserve necessary to cover all the payments of a life annuity is a fundamental issue for insurance companies, as well as governments and public or private entities paying their own pensions. In actuarial science, that reserve is calculated as the sum of the expected present values of the possible payments. This calculation involves factors such as the probability of living, inflation, ages, the amount of the payment and the discount rate called “the technical interest rate”. The estimation of the amount of these reserves directly affects the stability and coverage of pension systems and thus the quality of life of millions of people.

Battocchio & Menoncin (2002) review the existing literature on pension plans and conclude that there are two types. On the one hand, there are defined-benefit plans, where the sponsor previously determines the benefits and the contributions adjust in order to maintain the balance of the fund. The second type is the defined-contribution plans, in which the contributions are fixed and the future benefits depend on the returns of the portfolio. In addition,
the authors affirm that many of the pension plans offered based on defined-contribution schemes, transferring risk to the employees.

Regarding the design of pension alternative models or structural changes, Arellano (2018) perform an exploratory analysis of the design of non-contributory or social pensions implemented by local governments in Colombia, Spain and Mexico. Josa-Fombellida y Rincón-Zapatero (2019) study “the optimal management of an aggregated overfunded pension plan of defined benefit type as a two-player noncooperative differential game” and D’Amato, Di Lorenzo and Sibillo (2017) propose a new contract with profit participation, which consists in a deferred life annuity with variable benefits changing according with two dynamic financial elements: the periodic financial result of the invested fund year by year and the first order financial technical base.

On the other hand, Krebs (2019) study the excessive saving in pension systems and the inefficient capital allocations, specifically in Germany. For his part, Szabó (2017) use modelling and simulation to present a possible scenario of changes in the pension benefits in Hungary.

As described by Bodie & Crane (1999), the popularity of defined-contribution plans has two main reasons: i) the employee knows at any time the value of his retirement account and ii) this type of plan is much easier to manage by the sponsor than the defined-benefit.

In actuarial science relating to life annuities, it is generally assumed that the technical interest rate at which reserves are calculated to cover payments is equal to the expected rate of return of the portfolios where the reserves are invested, as indicated by Holsboer (2000) and Arango et al. (2013). This assumption is transversal to defined-contribution and defined-benefit plans, this is why the problem of choosing the best investment strategy to manage pension funds becomes extremely important in the management of such portfolios (Charupat & Milevsky, 2002).

Consider as a discount rate for the calculation of reserves the expected rate of return of the portfolio, it is a normal practice but not necessarily correct. According to Merton (2012), this situation could lead the portfolio managers to take greater risk in order to have greater profitability and at the same time decrease their pension liabilities.

Multiple investigations have addressed to the study of financial phenomena considering their stochastic dynamics. Some seminal works with this type of approach are
Black & Scholes (1973), Merton (1973), Vasicek (1977), and Cox Ingersoll & Ross (1985a) and (1985b). Other works that generalize the Itô processes found in Grinols & Turnovsky (1993) and Schmedders (1998).

Battocchio & Menoncin (2002) specify the behavior of the stochastic variables involved using the most common functional forms adopted in the literature.

This paper demonstrates that assuming the rate at which reserves are calculated to cover the payment of life annuities equals the expected rate of return of the portfolios in which they are invested is not an appropriate situation, if it is abandoned the assumption of constant interest rates and included within the analysis the stochastic nature of portfolio value. In addition, a procedure is presented to find the technical interest rates that make the probability of default on each of the income payments to be zero.

After this introduction, in section 2, the model is presented; in section 3, an application is shown, and section 4 presents some conclusions.

**Model**

*Reserve needed for life annuity*

Let \( f(w) \) be a survival function that indicates the probability that the age of death \( (W) \) of a particular person is greater than a \( w \) age. Thus:

\[
f(w) = P(W > w)
\]  
(1)

We define \( \tau p_w \) as the probability that a person of age \( w \) does not die in the next \( \tau \) years and \( \tau q_w \) as the probability of dying in the same period. According to this definition, we have:

\[
\begin{align*}
\tau p_w &= \frac{P(W > w + \tau | W > w)}{P(W > w)} \\
\tau p_w &= \frac{P(W > w \cap W > w + \tau)}{P(W > w)} \\
\tau p_w &= \frac{P(W > w + \tau)}{P(W > w)}
\end{align*}
\]
\[
\tau p_w = \frac{f(w + \tau)}{f(w)} \quad (2)
\]

As therefore,
\[
\tau q_w = 1 - \frac{f(w + \tau)}{f(w)} = \frac{f(w) - f(w + \tau)}{f(w)} \quad (3)
\]

The reserve necessary for the payment of a life annuity shall be the sum of the expected present values of the possible payments:
\[
R_N = E \left( \sum_{n=0}^{z-w} (Payment_n)(1 + i)^{-n} \right)
\]
\[
= E(Payment_0)(1 + i)^{-0} + E(Payment_1)(1 + i)^{-1} + E(Payment_2)(1 + i)^{-2} + \cdots + E(Payment_{z-w})(1 + i)^{-(z-w)}
\]
\[
R_N = (D_0p_w)(1 + i)^{-0} + (D_1p_w)(1 + i)^{-1} + (D_2p_w)(1 + i)^{-2} + \cdots + (D_{(z-w)}p_w)(1 + i)^{-(z-w)}
\]
\[
+ i)^{-(z-w)} \quad (4)
\]

where:
- \(D_{\tau}\): Amount of the allowance at an instant \(\tau\)
- \(i\): Effective interest rate.
- \(w\): Current age of the insured.
- \(z - w\): Maximum time that the insured can live.
- \(R_N\): Reserve necessary for the payment of a life annuity.

For convenience, this article assumes a life annuity with an indemnity that increases in the same percentage as the general price level and that initially corresponding to a value \(D\) in the event of non-death and zero (0) in the event of death. Life annuities with the assumed characteristics are the most usual, as they allow the beneficiary to maintain purchasing power by transferring the market risk.
Assuming that $D_T$ corresponds to a present value $D$ updated with the percentage change in the general price level ($\rho$), the necessary reserve expressed as:

$$R_N = \left( \sum_{n=0}^{\Omega-w} [D(1 + \rho)^n p_w(1 + r)^{-n} (1 + i)^{-n}] \right)$$

$$= [D(1 + \rho)^0 p_w (1 + i)^{-0} + D(1 + \rho)^1 p_w (1 + i)^{-1} + D(1 + \rho)^2 p_w (1 + i)^{-2} + \cdots + D(1 + \rho)^{(\Omega-w)} p_w (1 + i)^{-(\Omega-w)}]$$

Since the effective interest rate ($i$) can be considered as the combination of the percentage change in the general price level ($\rho$) and the real interest rate ($r$), then:

$$R_N = \left( \sum_{n=0}^{\Omega-w} [D(1 + \rho)^n p_w (1 + \rho)^{-n} (1 + r)^{-n}] \right)$$

$$= D(1 + \rho)^0 p_w (1 + i)^{-0} (1 + r)^{-0} + D(1 + \rho)^1 p_w (1 + i)^{-1} (1 + r)^{-1} + D(1 + \rho)^2 p_w (1 + i)^{-2} (1 + r)^{-2} + \cdots + D(1 + \rho)^{(\Omega-w)} p_w (1 + i)^{-(\Omega-w)} (1 + r)^{-(\Omega-w)}$$

In continuous time,

$$R_N = D \int_0^{\Omega-w} \tau p_w e^{-r \tau} d\tau$$

Such as,

$$\lim_{k \to \infty} \left( 1 + \frac{1}{k} \right)^k = e$$

$$FV = PV \lim_{k \to \infty} \left( 1 + \frac{i(k)}{k} \right)^{k \tau} = PV e^{i(\infty) \tau}$$

$$FV = PV e^{\delta \tau}$$

where:
\( k \): Frequency of capitalization of interest compost model with nominal interest rate.

\( FV \): Future Value.

\( PV \): Present Value.

**Probability of default on payment**

Let \( V_t \) be the value of the portfolio necessary to support the future payment \( D_T \), with \( 0 \leq T \leq (\Omega - w) \), which will be realized as long as the person of age \( w \) survives at least \( T - t \) years more, therefore, the fund value corresponds to the actuarial present value:

\[
V_t = D_{(T-t)} p_w e^{-r*(T-t)}
\]

(11)

where \( r \) is the real technical interest rate, \( (T-t)p_w \) is the probability that person of age \( w \) survive at least \( T - t \) years more and \( D_T = D e^{-\rho(T-t)} \), therefore \( D = D_T e^{-\rho(T-t)} \).

Assume that the portfolio value \( V_t \) is driven by the following Stochastic Differential Equation (SDE):

\[
dV_t = \mu_v V_t dt + \sigma_v V_t dW_t
\]

(12)

Note that at time \( T \):

If \( V_T < D_T \), failure to pay, at least partially, and there is no surplus \( E_T \) in the portfolio that supports the life annuity, \( E_T = 0 \).

If \( V_T \geq D_T \), the obligation is fulfilled, thus \( E_T > 0 \).

It is concluded that the surplus \( E_T \) can be modeled as:

\[
E_T = \max(V_T - D_T, 0)
\]

(13)

It is known that a call option whose future payment is obtained from the following expression:

\[
c = \max(S_T - X, 0)
\]

(14)

where \( S_T \) is the price of the asset at time \( T \) and \( X \) is the exercise price of the option.

By the equations of Black & Scholes (1973) and Merton (1973), the price at time \( t \) of this option can be calculated with:

\[
c_t = S_t N(d_1) - X e^{-\delta(T-t)} N(d_2)
\]

(15)
where:

\[ d_1 = \frac{\ln \left( \frac{S_t}{X} \right) + \left( \delta + \frac{1}{2} \sigma^2 \right) (T - t)}{\sigma \sqrt{T - t}} \]  \hspace{1cm} (16)\\

\[ d_2 = d_1 - \sigma \sqrt{T - t} \]  \hspace{1cm} (17)

In accordance with the previous development, the present value of surplus \( E_t \) is:

\[ E_t = V_t N(d_1) - D_T e^{-\mu_v(T-t)} N(d_2) \]  \hspace{1cm} (18)

where:

\[ d_1 = \frac{\ln \left( \frac{V_t}{D_T} \right) + \left( \mu_v + \frac{1}{2} \sigma_v^2 \right) (T - t)}{\sigma_v \sqrt{T - t}} \]  \hspace{1cm} (19)

\( N(y) \) is the probability that a normally distributed variable with zero mean (0) and standard deviation one (1) is less than \( y \).

As \( V_t = D_T - t p_w e^{-r(T-t)} \) and \( D_T = D e^{\rho(T-t)} \), therefore:

\[ d_1 = \frac{\ln \left( D_T / p_w e^{-r(T-t)} \right) + \left( \mu_v + \frac{1}{2} \sigma_v^2 \right) (T - t)}{\sigma_v \sqrt{T - t}} \]  \hspace{1cm} (20)

Following Merton (1974), it is possible to calculate in the following way the probability of default, totally or partially, with the payment \( D_T \):

It's known that:

\[ E_t = e^{-\mu_v(T-t)} \int_{-\infty}^{\infty} E_T f_{V_T | V_t}(V | V_t) dv \]

\[ E_t = e^{-\mu_v(T-t)} \int_{-\infty}^{\infty} \max(v - D_T, 0) f_{V_T | V_t}(V | V_t) dv \]
\[ E_t = e^{-\mu v(T-t)} \int_{D_T}^\infty (v - D_T) f_{V_T|V_t}(V|V_t)dv \]

\[ E_t = e^{-\mu v(T-t)} \int_{D_T}^\infty v f_{V_T|V_t}(V|V_t)dv - D_T e^{-\mu v(T-t)} \int_{(v>D_T)}^\infty f_{V_T|V_t}(V|V_t)dv \]

\[ E_t = e^{-\mu v(T-t)} E\left[V_T|V_T > D_T\right] - D_T e^{-\mu v(T-t)} \mathbb{P}[V_T > D_T|V_t] \]  \hspace{1cm} (21)

where \( f_{V_T|V_t}(V|V_t) \) is the probability density function of \( V_T \), conditional to the initial value \( V_t \).

As \( E_t = V_t N(d_1) - D_T e^{-\mu v(T-t)} N(d_2) \), then:

\[ \mathbb{P}[V_T > D_T|V_t] = N(d_2) \] \hspace{1cm} (22)

Therefore, the probability of default will be:

\[ \mathbb{P}[V_T < D_T|V_t] = 1 - N(d_2) \] \hspace{1cm} (23)

And for the latter to be zero, it is necessary that \( N(d_2) = 1 \).

\textit{Estimation of the technical interested rate}

Bearing in mind that \( E_t = V_t N(d_1) - D_T e^{-\mu v(T-t)} N(d_2) \), can be deduce for the Itô lemma that:

\[ dE_t = \left( \frac{\partial E_t}{\partial t} + \mu V_t \frac{\partial E_t}{\partial V_t} + \frac{1}{2} \sigma^2_v V_t^2 \frac{\partial^2 E_t}{\partial V_t^2} \right) dt + \sigma_v V_t \frac{\partial E_t}{\partial V_t} dW_t \] \hspace{1cm} (24)

Assuming that the surplus \( E_t \) is driven by the process \( dE_t = \mu S_E t dt + \sigma S_E dW_t \), then:

\[ \mu S_E dt + \sigma S_E dW_t \]

\[ = \left( \frac{\partial E_t}{\partial t} + \mu V_t \frac{\partial E_t}{\partial V_t} + \frac{1}{2} \sigma^2_v V_t^2 \frac{\partial^2 E_t}{\partial V_t^2} \right) dt + \sigma_v V_t \frac{\partial E_t}{\partial V_t} dW_t \] \hspace{1cm} (25)

Since the stochastic components must be equal:

\[ \sigma S_E = \sigma_v V_t \frac{\partial E_t}{\partial V_t} \] \hspace{1cm} (26)

As \( E_t = V_t N(d_1) - D_T e^{-\mu v(T-t)} N(d_2) \), then \( \frac{\partial E_t}{\partial V_t} = N(d_1) \), therefore:

\[ \sigma S_E = \sigma_v V_t N(d_1) \] \hspace{1cm} (27)
\[ E_t = \frac{\sigma_v}{\delta S} V_t N(d_1) \]  

Replacing this last expression in \( E_t = V_t N(d_1) - D_T e^{-\mu_v(T-t)} N(d_2) \) is obtained:

\[ \frac{\sigma_v}{\delta S} V_t N(d_1) = V_t N(d_1) - D_T e^{-\mu_v(T-t)} N(d_2) \]  

\( \hat{\delta}_S \) is observable, \( \hat{\mu}_v \) and \( \hat{\sigma}_v \) they are also if a portfolio has already been constituted to guarantee the life annuity, consequently it is possible to solve the following equation for the real technical interest rate \( r \); subject to the constraint that the probability of default is zero,

\[ \mathbb{P}\{V_T < D_T | V_t\} = 1 - N(d_2) = 0, \text{ therefore:} \]

\[ 0 = V_t N(d_1) \left( 1 - \frac{\hat{\delta}_v}{\hat{\delta}_S} \right) - D_T e^{-\hat{\mu}_v(T-t)} \]  

which is the same:

\[ 0 = D_{(T-t)} p_w e^{-r(T-t)} N(d_1) \left( 1 - \frac{\hat{\delta}_v}{\hat{\delta}_S} \right) - D_T e^{-\hat{\mu}_v(T-t)} \]  

where (equation 32):

\[ d_1 = \frac{\ln \left( \frac{p_w e^{-r(T-t)} N(d_1)}{\delta S} \right) + \left( \hat{\mu}_v + \frac{1}{2} \sigma_v^2 \right)(T-t)}{\sigma_v \sqrt{T-t}} \]  

\[ d_2 = d_1 - \hat{\delta}_v \sqrt{T-t} \]  

Assuming that \( V_t = D_{(T-t)} p_w e^{-r(T-t)} \).

As \( D_T = D e^{\rho(T-t)} \), then

\[ 0 = D_{(T-t)} p_w e^{-r(T-t)} N(d_1) \left( 1 - \frac{\hat{\delta}_v}{\hat{\delta}_S} \right) - D e^{\rho(T-t)} e^{-\hat{\mu}_v(T-t)} \]  

\[ 0 = D_{(T-t)} p_w e^{-r(T-t)} N(d_1) \left( 1 - \frac{\hat{\delta}_v}{\hat{\delta}_S} \right) - D e^{-\hat{\mu}_v(T-t) - \rho(T-t)} \]  

\[ 0 = (T-t) p_w e^{-r(T-t)} N(d_1) \left( 1 - \frac{\hat{\delta}_v}{\hat{\delta}_S} \right) - e^{-\hat{\mu}_v(T-t) - \rho(T-t)} \]  

Finally, removing the constraint \( \mathbb{P}\{V_T < D_T | V_t\} = 0 \), we have (equation 37):

\[ 0 = (T-t) p_w e^{-r(T-t)} N(d_1) \left( 1 - \frac{\hat{\delta}_v}{\hat{\delta}_S} \right) - e^{-\hat{\mu}_v(T-t) - \rho(T-t)} N(d_2) \]  

The above procedure allows finding a different real technical interest rate for each payment to be made. The full actuarial calculation, as approximation, would then be:

\[ V_t = D_0 p_w e^{-\hat{\rho}_0 t} + D_1 p_w e^{-\hat{\rho}_1 t} + D_2 p_w e^{-\hat{\rho}_2 t} + \ldots + D_{\Omega-w} p_w e^{-\hat{\rho}_{\Omega-w} (\Omega-w)} \]  

\[ (38) \]
Application

In Colombia, the Fondo de Pensiones de las Entidades Territoriales (FONPET) is a fund set up by the Ministry of Finance and Public Credit to support the payment of the pensions of former employees through Colombian territory. FONPET is defined as “a fund without legal status administered by the Ministry of Finance and Public Credit, which aims to collect and allocate resources to the accounts of the territorial entities and administer the resources through the autonomous patrimonies” (Dirección de Inversiones y Finanzas Públicas, 2017).

Today, FONPET manages approximately $16 000 USD millions, being the largest and most relevant fund in the country. The financial stability of departments, municipalities and other governmental entities, depend on the correct administration of that fund.

Figure 1 shows the evolution of the per unit return of FONPET from January 1, 2013 to November 21, 2018. In this period, its annual profitability was $\mu_v^*=6.01\%$ and its annual volatility $\sigma_v^*=1.77\%$.

\[
\tilde{\sigma}_s = \left[ \frac{\sum (L_t - \bar{L}_t)^2}{n - 1} \right]^{1/2} \sqrt{252}
\]
Where,

\[ L_t = \ln \left( \frac{E_t}{E_{t-1}} \right) \]

And the surplus is,

\[ E_t = \max \left( V_t - V_T e^{-\frac{\mu S}{252}(T-t)}, 0 \right) \]

If an inflation rate of 3% is assumed, the technical interest rate can be found by applying equation 36 and 32 as follows:

\[ 0 = (T-t)p_w e^{-r(T-t)} N(d_1) \left( 1 - \frac{1.77\%}{166.34\%} \right) e^{-\left(6.01\%-3\%\right)(T-t)} \]

\[ d_1 = \frac{\ln \left( (T-t)p_w e^{-(r+3\%)(T-t)} \right) + \left( 6.01\% + \frac{1}{2} (1.77\%)^2 \right) (T-t)}{1.77\% \sqrt{T-t}} \]

In Colombia, the retirement age in the public pension system is 62-year-old for men and 57-year-old for women. For the case of a 62-year-old man, the actuarial calculation should consider the technical interest rates shown in Figure 2. Under the traditional methodology, considering a portfolio's profitability of 6.01% and inflation rate of 3.0% per year, the technical annual interest rate used will be 3.01%. Under the perspective of multiple technical interest rates, only the flow of year 17 must be discounted at that interest rate.
Through the traditional methodology, the same technical interest rate is applied in each of the possible cash flows as it is in the case of Colombia. Now let’s considered annual payments of $12,000 USD through time, the value of the reserve will be $168,247 USD under the traditional methodology with a technical interest rate of 4% per year. Through the proposed methodology, the value of the reserve will be $222,408 USD. It is clearly observed that the inherent risk of this life annuity is underestimated by the current system.

To analyze scenarios in which the profitability of the reserve changes, a structure of coefficients of constant variation was assumed.

\[
\frac{\hat{\sigma}_v}{\hat{\mu}_v} = 0.293458 \\
\frac{\hat{\sigma}_s}{\hat{\mu}_v} = 27.654655
\]

Based on this structure, scenarios of true profitability minus 3%, 2% and 1% and true profitability plus 1%, 2% and 3% were simulated. The results are shown in figure 3.
As shown in Table 1, adding an additional 2% to the profitability of the fund would result in a decrease in the actuarial reserve required. On the other hand, it can be seen the risk is being underestimated since the client is being required to pay $168,247 USD, while the calculations estimate this reserve at US $222,408 USD.

Table 1
Profitability vs Actuarial Reserve for a 62-year-old man

<table>
<thead>
<tr>
<th>Return</th>
<th>Actuarial Reserve</th>
<th>Change (%) Today vs. Scenarios</th>
<th>Change (%) (add 1% in return)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today (rtech=4%)</td>
<td>168,247</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-3% in return</td>
<td>503,858</td>
<td>199.48%</td>
<td></td>
</tr>
<tr>
<td>-2% in return</td>
<td>369,916</td>
<td>119.87%</td>
<td>-26.6%</td>
</tr>
<tr>
<td>-1% in return</td>
<td>362,162</td>
<td>67.71%</td>
<td>-23.7%</td>
</tr>
<tr>
<td>True return</td>
<td>222,408</td>
<td>32.19%</td>
<td>-21.2%</td>
</tr>
<tr>
<td>+1% in return</td>
<td>180,238</td>
<td>7.13%</td>
<td>-19.0%</td>
</tr>
<tr>
<td>+2% in return</td>
<td>149,381</td>
<td>-11.21%</td>
<td>-17.1%</td>
</tr>
<tr>
<td>+3% in return</td>
<td>126,522</td>
<td>-24.80%</td>
<td>-15.3%</td>
</tr>
</tbody>
</table>

Source: Own elaboration with Excel 2016.
Another effect that is observed in the Table 1, an increase in profitability produces a decrease in the actuarial reserve required.

For the case of a 57-year-old woman, the results are presented in Figure 4 and Table 2.

![Figure 4. Technical interest rates of a 57-year-old woman. Source: Own elaboration with Excel 2016.](image)

<table>
<thead>
<tr>
<th>Return</th>
<th>Actuarial Reserve</th>
<th>Change (%) Today vs. Scenarios</th>
<th>Change (%) (add 1% in return)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today (rtech=4%)</td>
<td>246 555</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-3% in return</td>
<td>510 368</td>
<td>107.00%</td>
<td>-26.6%</td>
</tr>
<tr>
<td>-2% in return</td>
<td>374 765</td>
<td>52.00%</td>
<td>-26.6%</td>
</tr>
<tr>
<td>-1% in return</td>
<td>287 572</td>
<td>16.64%</td>
<td>-23.3%</td>
</tr>
<tr>
<td>True return</td>
<td>225 309</td>
<td>-8.62%</td>
<td>-21.7%</td>
</tr>
<tr>
<td>+1% in return</td>
<td>182 947</td>
<td>-25.80%</td>
<td>-18.8%</td>
</tr>
<tr>
<td>+2% in return</td>
<td>152 029</td>
<td>-38.34%</td>
<td>-16.9%</td>
</tr>
<tr>
<td>+3% in return</td>
<td>128 521</td>
<td>-47.87%</td>
<td>-15.5%</td>
</tr>
</tbody>
</table>

Source: Own elaboration with Excel 2016.
In this case, table 2 shows that the current real yield produces a reduction of 8.62% in the actuarial reserve required compared to what is required. In this case, the risk is being overestimated.

It is also observed that an additional real profitability of 1% would produce a 25.8% decrease in the actuarial reserve required.

If it is a group life annuity, in which if the owner dies another person will inherit the payments, the equations for this system are:

\[
0 = \left[ (T-t)p_{x_1,x_2} \right] e^{-r(T-t)} N(d_1) \left( 1 - \frac{\hat{\sigma}_v}{\hat{\sigma}_s} \right) - e^{-((\hat{\mu}_v-\rho)(T-t))} N(d_2)
\]

\[
d_1 = \frac{\ln \left( \left[ (T-t)p_{x_1,x_2} \right] e^{-(r+p)(T-t)} \right) + \left( \hat{\mu}_v + \frac{1}{2} \hat{\sigma}_v^2 \right) (T-t)}{\hat{\sigma}_v \sqrt{T-t}}
\]

Where, \( (T-t)p_{x_1,x_2} \) is the probability that, of a group consisting of two people with information \( x_1 \) and \( x_2 \) respectively, at least one (1) of them will survive, at least, \( T - t \) years more.

If a couple is supposed to be composed of a 57-year-old woman and a 62-year-old man, the results shown in Figure 5 and Table 3 are obtained.
In this case, using multiple technical interest rates makes it possible to reduce the cost of a life annuity granted by the national, departmental or municipal governments by more than 15%. This is done while maintaining the same profitability and volatility of the FONPET portfolio.

**Figure 5.** Multiple technical interest rates for a couple of a 57-year-old woman and a 62-year-old man. Source: Excel 2016.

**Table 3**
Profitability vs Actuarial Reserve for a couple of a 57-year-old woman and a 62-year-old man

<table>
<thead>
<tr>
<th>Return (rtech=4%)</th>
<th>Actuarial Reserve</th>
<th>Change (%) Today vs. Scenarios</th>
<th>Change (%) (add 1% in return)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today</td>
<td>262 095</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-3% in return</td>
<td>494 676</td>
<td>88.74%</td>
<td></td>
</tr>
<tr>
<td>-2% in return</td>
<td>366 366</td>
<td>39.78%</td>
<td>-25.9%</td>
</tr>
<tr>
<td>-1% in return</td>
<td>282 177</td>
<td>7.66%</td>
<td>-23.0%</td>
</tr>
<tr>
<td>True return</td>
<td>221 951</td>
<td>-15.32%</td>
<td>-21.3%</td>
</tr>
<tr>
<td>+1% in return</td>
<td>180 714</td>
<td>-31.05%</td>
<td>-18.6%</td>
</tr>
<tr>
<td>+2% in return</td>
<td>150 522</td>
<td>-42.57%</td>
<td>-16.7%</td>
</tr>
<tr>
<td>+3% in return</td>
<td>127 645</td>
<td>-51.30%</td>
<td>-15.2%</td>
</tr>
</tbody>
</table>

Source: Own elaboration with Excel 2016.
Increasing the portfolio's 1% return would allow a 31% decrease in the cost of that particular annuity.

**Conclusions**

This document shows a procedure to calculate the technical interest rates that should be used to discount each possible payment in the actuarial reserve necessary for a life annuity. This method is constructed from the model of Merton (1974) to estimate the probabilities of default in corporate bonds. It is an alternative to the traditional option of assuming a constant and equal discount rate for all cash flows and which is usually the expected long-term yield of the portfolio, as is shown in Bowers, Gerber, Hickman, Jones and Nesbitt (1997), Arango (2013), D’Amato, V., Di Lorenzo, E., & Sibillo, M. (2017) and Szabó (2017).

In addition, the effects of its application to the Pension Fund of Territorial Entities (FONPET) in Colombia were analyzed. The effect of using multiple technical interest rates on actuarial reserves calculated for a 62-year-old man, a 57-year-old woman and a couple with the same sexes and ages previously described was estimated.

In the case of the 62-year-old man, using this method with the rates of profitability and historical volatility of the FONPET, will imply an increase of 32.19% in the actuarial reserve required. Applying it to the 57-year-old woman will imply a decrease of 8.62% and applying it to an annuity of two lives (62-year-old man and 57-year-old woman) will have an effect of -15.32%.

Future research can deal with the effects that this method would have on the reduction of Colombia's fiscal deficit as a particular case.

**References**


