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Contaduría y Administración

Accounting & Management

Contaduría y Administración 66 (1), 2021, 1-31

Investment with Markov-Switching GARCH models: A comparative study of Mexico and Argentina

Inversión con modelos Markov-Switching GARCH: un estudio comparativo entre México y Argentina

Oscar V. De la Torre-Torres*

Universidad Michoacana de San Nicolás de Hidalgo

Received July 20, 2019; accepted February 18, 2020
Available online November 17, 2023

Abstract

In the present paper I test the benefits, for active portfolio management purposes, of using two-regime Markov-Switching (MS) models with GARCH variance. This, with either a Gaussian or t-Student homogeneous likelihood function, in the Buenos Aires and in the Mexican Stock Exchanges. By performing 996 weekly simulations from January 2000 to January 2019 in each MS model, I tested the next investment strategy for a U.S. dollar-based investor: 1) to invest in the risk-free asset if the probability of being in the high-volatility regime at t+1 is higher than 50% or 2) to do it in an equity index otherwise. The results suggest that the t-Student MS-GARCH model is the best option to generate alpha in Argentina and the constant variance gaussian one in Mexico. This, against a "buy and hold" investment strategy.

JEL Code: C580, G11, G170
Keywords: Markov-Switching GARCH; markov chain processes; active portfolio management; buenos aires stock exchange; mexican stock exchange; frontier markets; computational finance; risk management

* Corresponding author.
E-mail address: odelatorre@umich.mx (O. V. De la Torre Torres).
Peer Review under the responsibility of Universidad Nacional Autónoma de México.

http://dx.doi.org/10.22201/fca.24488410e.2020.2657

http://dx.doi.org/10.22201/fca.24488410e.2020.2657

Resumen

En este artículo se estudia el empleo de modelos markovianos con cambio de régimen (Markov-Switching) de dos regímenes, varianza GARCH y con funciones de verosimiltud gaussiana o t-Student homogéneas entre regímenes. Esto para administrar activamente portafolios en la bolsa de Buenos Aires y la Bolsa Mexicana de Valores. Al realizar 996 simulaciones semanales de enero del 2000 a enero del 2019, se ejecutó la siguiente estrategia de inversión para un portafolio denominado en dólares de los EEUU: 1) invertir en el activo libre de riesgo si la probabilidad de estar en el régimen de alta volatilidad en t+1 es mayor a 50% o 2) invertir en el índice accionario en caso contrario. Los resultados sugieren que emplear modelos MS-GARCH t-Student en una administración activa lleva a un mejor desempeño en el caso argentino y los modelos MS con varianza constante y función gaussiana en el mexicano. Esto en comparación con una estrategia pasiva tipo "comprar y mantener".

Código JEL: C580, G11, G170
Palabras clave: markov-switching GARCH; cadenas markovianas; administración activa de portafolios; bolsa de comercio de buenos aires; bolsa mexicana de valores; mercados frontera; finanzas computacionales; administración de riesgos

Introduction

One of the most sensitive tasks in financial practice, specifically in investment and risk management, is adequately quantifying the statistical parameters used for decision-making. Another sensitive and related activity is the proper calculation or inference of the appropriate moment to invest or disinvest in a portfolio. As will be seen in this paper, using Markovian models with regime switching (Markov-Switching models or MS from now on) will be very useful to determine the appropriate moments for investment and disinvestment. This is based on calculating a probability ξ_(s=2,t) of being in a period, regime, or state of nature known as "high volatility." By determining this probability, both an expected return and a level of risk exposure are obtained as output parameters, which will be part of calculating the likelihood above and are also central parameters or inputs for making investment decisions in financial practice.

In professional practice and academia, methods such as the arithmetic mean or the exponential phase have been used to quantify the level of return expected by the investor. In a complementary manner and as will be seen in the review of the literature that motivates the present work, multiple conditional mean methods have been suggested for purposes of computational efficiency or to give greater robustness to the quantification of these parameters (Alexander, 2002; Ang & Bekaert, 2002a; Sharpe, 1963, 1964), which quantify a conditional mean return, given the value that a factor or group of factors can have in t

Other proposals or extensions to quantifying expected returns are given using ARMA[1] or ARMAX models. In the first case, it is established that the stochastic process that generates the time series of the returns of an asset or security r_t depends, given different situations specific to the modeled asset, on past values (lags) of the returns as well as on lags in the residuals ε_t

$$r_t = \alpha + \sum_{p=1}^{P} \beta_p \cdot r_{t-p} + \sum_{q=1}^{Q} \gamma_p \cdot \varepsilon_{t-1} + \varepsilon_t \tag{1}$$

The model in (1) is known as an ARMA process (previously defined in footnote 1), noting that the second term of the expression is known as the AR or autoregressive term, which measures the impact of past values ($r_{i,t-p}$) on the current returns level ($r_{i,t}$). The third term in (**¡Error! No se encuentra el origen de la referencia.**) is known as MA or moving average, which measures the impact that past values of the residuals ($\varepsilon_{i,t-q}$) of (**¡Error! No se encuentra el origen de la referencia.**) have on the value of $r_{i,t}$.

As a starting point for quantifying returns in this work, the practice of asset valuation and the literature related to Financial Econometrics will be followed, consisting of quantifying r_t with the method of continuously compounded returns, which is based on the current price $\left(P_{i,t}\right)$ and the past price $(pi_{,t-1})$ of the asset, index, currency or commodity under analysis:

$$r_{i,t} = \ln\left(P_{i,t}\right) - \ln\left(p_{i,t-1}\right) \tag{2}$$

The above enables the time series in r_t to be stationary[2], given the calculation of the first difference of the logarithm of the price. Starting from this situation, (**¡Error! No se encuentra el origen de la referencia.**) is the functional form of an ARMA model, whose forecasts at $t+1$ are integrated[3] into the current price (P_t). This to forecast its price in p_{t+1}.

Starting from the quantification of the expected return with one of the previously described methods, it is observed that the level of risk exposure that began to be approximated, in the early historical

[1]For exposition purposes, the acronyms ARMA (Auto Regressive Moving Average) and ARIMA (Auto Regressive Integrated Moving Average) will be used.

[2]Stationarity is a property that the r_t time series must have in order to be able to perform econometric analysis with them. This means that the location (mean) and scale (variance) parameters must be constant over time or, if not possible, stable in their value over time.

[3]Hence the essence and name of the ARIMA (p,I,q) model that applies for p_t, given the ARMA model in (1).

http://dx.doi.org/10.22201/fca.24488410e.2020.2657

stages of contemporary financial practice, with the conventional variance ($\sigma_{i,t}^2 = \sum_{t=1}^{T}(r_{i,t} - \mu_i) \cdot N^{-1}$) is of similar importance. The limitation of this calculation method is that it has a constant value over time. Nevertheless, with the proposals of Engle (1982) and Bollerslev (1987), significant progress was made in quantifying this parameter since the variance can be estimated as a changing parameter, which is an essential characteristic of GARCH[4] models and something typical of financial time series that do not have constant volatilities or variances over time:

$$\sigma_{i,t}^2 = \sigma_0 + \sum_{p=1}^{P} \beta_p \cdot \varepsilon_{i,t-p}^2 + \sum_{q=1}^{Q} \gamma_p \cdot \sigma_{i,t-q}^2 + \nu_t \tag{3}$$

In the previous expression and as mentioned, the value of the variance σ_t^2 at t is quantified using the quadratic values of the residuals, which are determined with the mean or expected return ($\varepsilon_{i,t-p}^2$), as well as the past values of the variance ($\sigma_{i,t-q}^2$) estimated in (**¡Error! No se encuentra el origen de la referencia.**). The second and third terms in (**¡Error! No se encuentra el origen de la referencia.**) are ARCH and GARCH. Given this, the model is said to be an ARCH one when (**¡Error! No se encuentra el origen de la referencia.**) has this functional form: $\sigma_t^2 = \sigma_0 + \sum_{p=1}^{P} \beta_p \cdot \varepsilon_{i,t}^2 + \nu_t$. Alternatively, it is known as GARCH when (**¡Error! No se encuentra el origen de la referencia.**) is expressed in full.

The method of quantifying risk or variances dynamically over time using GARCH models allowed great advances in the financial industry. More specifically, progress was made in asset valuation and risk management. This is because, in times of crisis or high financial volatility (price fluctuations), high variances are measured, and in “normal” or low volatility periods, low levels of volatility are measured.

Based on this perspective of the GARCH models, multiple extensions to the GARCH models have been proposed. The exponential model of asymmetric effects EGARCH by Nelson (1991) and the GJR-GARCH with negative residuals leverage by Glosten, Jaganathan, and Runkle (1993) are among the most widely used and recognized. There are other extensions with different likelihood functions (which will not be mentioned here, given the breadth of the review).

A useful property of the GARCH model, such as the one presented in (**¡Error! No se encuentra el origen de la referencia.**) is the fact that the sum $\sum_{p=1}^{P} \beta_p + \sum_{q=1}^{Q} \gamma_p$ leads to a concept known as “persistence,” which implies that, if $\sum_{p=1}^{P} \beta_p + \sum_{q=1}^{Q} \gamma_p \approx 1$ in (**¡Error! No se encuentra el origen de la referencia.**), high volatility levels will “persist” for long periods over time. This situation can occur in

[4] Acronym for Generalized Auto Regressive Conditional Heteroskedasticity.

financial time series such as those generated with (**¡Error! No se encuentra el origen de la referencia.**). As one of the multiple explanations given for this result, Dueker (1997), Lamoureoux and Lastrapes (1990), Hamilton and Susmel (1994), Klaassen (2002), and Hass, Mitnik, and Paolella (2004) propose that the high persistence is because the stochastic process of the time series should not be modeled in a unimodal way. That is, with one mean and standard deviation (and, therefore, a single probability function) but with multiple means and standard deviations, characteristic of a multi-modal probability function. This leads to conceiving the time series as one that has $s-1$ (with $s=1.2,\ldots,S$) structural changes leading to the presence of not one, but S regimes or states of nature in the behavior of r_t.

For this reason, and thanks to Hamilton's proposals (1989, 1994), the expected return and risk exposure (variance) can be approximated utilizing an S-state Markovian shift (MS) model. Given the above, the ARMA model in (**¡Error! No se encuentra el origen de la referencia.**) can be extended as follows:

$$r_{i,t}=\alpha_s+\sum_{p=1}^{P}\beta_{p,s}\cdot r_{i,t-p}+\sum_{q=1}^{Q}\gamma_{p,s}\cdot\varepsilon_{t-1}+\varepsilon_t \tag{4}$$

Given the parameter vector $\theta=[\alpha_s,\beta_s,\gamma_s]$, the MS model makes it possible, as an output parameter, to infer the probability $\xi_{s,t}$ of being in regime S at t. In addition, the transition probabilities $(\pi_{i,j}=P(s_t=i|s_{t-1}=j,\theta,r_t),\theta=[\alpha_s,\beta_s,\gamma_s,\sigma_s^2,\xi_{s,t}])$ estimate the likelihood of transitioning from regime $s=i$ at t, to regime $s=j$ at $t+1$. As can be seen, this behavior is typical of a latent Markovian chain and is not directly observable by the analyst. Given the above, the matrix representation of these transition probabilities is summarized in what is known as the transition probability matrix $\mathbf{\Pi}$:

$$\mathbf{\Pi}=\begin{bmatrix}\pi_{i,i} & \cdots & \pi_{j,i}\\ \vdots & \vdots & \vdots\\ \pi_{i,j} & \cdots & \pi_{j,j}\end{bmatrix} \tag{5}$$

A limitation of the MS model as given in (**¡Error! No se encuentra el origen de la referencia.**) is that, in its original form, it assumes that each regime's variance (read standard deviation) is constant over time. Given this, an extension to the MS model is made by Hamilton and Susmel (1994), Klaassen (2002), and Haas, Mitnik and Paolella (2004), in which the GARCH model in (**¡Error! No se encuentra el origen de la referencia.**), is now a proper one of S regimes. In other words, a GARCH model with parameters for each regime (MS-GARCH):

$$\sigma_t^2 = \sigma_{0,s} + \sum_{p=1}^{P} \beta_{p,s} \cdot \varepsilon_{t-p}^2 + \sum_{q=1}^{Q} \gamma_{p,s} \cdot \sigma_{t-q}^2 + \nu_t \tag{6}$$

Given the dynamic nature of MS models and the possibility of inferring smoothed and transition probabilities for S regimes, many applications of these models have been studied. The use of MS models in investment decision-making is the application of interest in this paper and is developed with the original proposal of Brooks and Persand, followed by Ang and Bekaert (2002a, 2004). These authors suggest the use of MS models in UK stock indices decision-making and the use of MS models to manage internationally diversified portfolios. These two works were followed by those of Kritzman, Page, and Turkington (2012), Hauptmann *et al*. (2014), and De la Torre, Galeana, and Alvarez-Garcia (2018), the latter being an application in both developed and an emerging[5] (Mexico) stock markets.

As will be seen in the review of the literature that motivates this paper, there are some areas of opportunity that the present work seeks to address:

1. Previous studies on the benefits of MS models in the investment decision-making process do not study the use of MS-ARCH or MS-GARCH models in the investment decision-making process.

2. Among the previous works focused on investment decision-making with MS models, only one focuses on emerging and Latin American countries (Mexico), leaving the opportunity to study its extension to other Latin American markets such as Argentina. This is through the MSCI Argentina index.

3. In different previous reviews, little has been studied on the benefit or use of MS, MS-ARCH or MS-GARCH models in the countries of interest, not only in investment areas but in modeling in general. The works of Camacho and Pérez-Quirós (2014), Cabrera *et al*. (2017) and Sosa, Ortiz, and Cabello (2018) carry out interesting modeling with two and three regimes. Specifically for this paper, Cabrera *et al*. (2017) is the only study using MS-GARCH models in Latin America. Given this, the aim is to extend its results to the application and use of MS-ARCH and MS-GARCH models in the region, especially regarding their application in investment decision-making.

[5] From November 19, 2001, until November 12, 2018, Argentina's sovereign credit rating was below D (according to the Standard & Poors scale), which is why, in most of the simulation performed, its equity markets had the "frontier" classification. The definition of the terms developed market, emerging market, and frontier market are established in the Global Industry Classification Standard (GICS), developed jointly by Morgan Stanley Capital International (MSCI) and Standard & Poors Dow Jones (S&P). This is in order to have market indices with an aggregate classification by country, sector, and type of value. For further information on the typology and the classification criteria used, please refer to MSCI Inc. (2018).

http://dx.doi.org/10.22201/fca.24488410e.2020.2657

4. Nothing has been reviewed or written regarding using MS, MS-ARCH, or MS-GARCH models in the investment decision-making process in securities markets classified as "frontier markets." This is done by Morgan Stanley Capital International (MSCI Inc., 2018), with the Buenos Aires Stock Exchange (Argentina) being a very representative example of this type of market and a very important one in Latin America's economic and financial activity.

The rationale for choosing these two countries is that these two economies are among Latin America's largest (in terms of GDP) (World Bank, 2019). On the other hand, Argentina was classified as a frontier country for most of the simulated period, while Mexico is considered an emerging country. Furthermore, the rationale for making a comparison between the Argentinean and Mexican stock markets is based on the fact that the former, as previously mentioned, is the most liquid and largest of the markets considered frontier in Latin America, and Mexico has the most traded currency in terms of hedging and "at the moment" or spot operations of all emerging countries (Bank for International Settlements, 2016).

Specifically, the aim is to demonstrate for these two markets that superior returns can be achieved for both emerging and frontier markets from the perspective of an investor whose portfolio is denominated in US dollars (USD). This is in comparison to a passive or "buy and hold" strategy. The argument is that this is achieved using MS-GARCH models within the investment decision-making process.

To achieve this objective, the following active investment strategy will be tested:

1. Invest in the market index of the simulated country if the investor expects to be in the normal or low volatility regime ($s = 1$) at $t + 1$ or
2. Invest in US risk-free assets if the investor expects to be in the ($s = 2$) high volatility regime in the abovementioned period.

Given this investment strategy and the motivations previously described, two working hypotheses are established to be proven here:

H_1: "The use of MS-GARCH models in an active investment strategy generates Alpha or additional returns compared to a passive buy-and-hold strategy in the Argentinean and Mexican stock markets."

H_2: "The use of MS-GARCH models in an active investment strategy generates a significant reduction in risk exposure in the Argentinean and Mexican stock markets. This is compared to a passive buy-and-hold strategy."

The reason for conducting the test from the perspective of a US dollar-denominated investor is to measure the attractiveness of employing the investment strategy to foreign investors, with institutional clients injecting the most trading and money flows into these markets (Refinitiv, 2018a). This position is based on the US dollar being the most traded currency internationally (Bank for International Settlements, 2016). Given these two facts, it is interesting to investigate whether investing in these two markets with the proposed investment strategy and from an investor's perspective in this currency is attractive.

Based on the previously described motivation and once the working hypotheses to be proven in this article have been established, this paper is structured as follows. In the next section, there will be a brief review of the literature. This is to contextualize the use of MS-GARCH models in the investment decision-making process and the proposed strategy within Financial Economics and portfolio theory. The third section will discuss, for introductory purposes for interested readers, the MS-GARCH model to be used in the investment decision-making process and the pseudocode that governed the simulations will be described. The fourth section presents a description of the input data and the results of tests on the relevance of using MS, MS-ARCH or MS-GARCH models in the investment decision-making process. The fifth and final section states the main conclusions and recommendations for future research.

Review of the Literature

Markov-Switching models and their relation to portfolio theory and investment management

A widely known result of Markowitz's (1959; 1952, 1956) proposal is that it is possible to estimate, as stated in the introduction, both the expected return and the level of risk exposure. The investor can quantify these two parameters within a profit function, subject to maximization. This generated a well-known evolution of the investment analysis and decision-making process to the extent that the financial industry has made significant progress in terms of volume traded, sophistication in client advice, and asset management. Despite this theoretical and practical advance, Markowitz's original proposal suffers from some limitations that have been studied and overcome in many aspects. The most important of these is that the vector of expected returns of each security being invested in and its corresponding covariance matrix are statistical parameters taken on a sample basis. That is, its value is determined based on the data in the sample mentioned above; its magnitude is subject to uncertainty. Accordingly, and thanks to the theories of Sharpe (1963, 1964) and Fama's work on informational efficiency (1965; 1963), two types of portfolio management can be distinguished. The first is passive management (Maggin, Tuttle, Pinto, & McLeavey, 2007), which consists of investing in a portfolio whose composition is identical to that of a market or benchmark portfolio. The second is active management, which consists of investing resources in a portfolio with a different asset allocation (investment levels) from that observed in the market index. This aims to generate extra or higher returns than the index (known as "additional returns" or Alpha).

http://dx.doi.org/10.22201/fca.24488410e.2020.2657

Multiple theories have been put forward regarding the uncertainty to which the parameters of the portfolio selection model refer. Some of them can be seen in the works of Michaud and Michaud (1989; 2008) and Jorion (1992) that use resampling techniques (Monte Carlo simulation or bootstrapping) or the use of Bayesian statistics such as the model of Black and Litterman (1992), from which multiple extensions have emerged (Xiao & Valdez, 2015). The latter cases seek to make a linear combination between the uninformed market parameters and the personal expectations or forecasts of one or several analysts, which implies a reduction in uncertainty regarding the vector of expected returns. Nevertheless, the resampling and Bayesian techniques for portfolio management suffer from a limitation (in their original form): They do not distinguish the value that the parameters can have in different states of nature or regimes, such as a low volatility state or regime ($s = 1$) and a high volatility state or regime ($s = 2$) where the fluctuation of returns tends to be greater than in the previous one[6]. Based on this and noting that the regime or state of nature may change from one period to another, parameters (mean vectors and covariance matrices) that are more representative of them should be used. As a result, two possible alternatives were developed to incorporate the uncertainty generated by the presence of S regimes of this nature. The first consists of using mixtures of probability functions such as the Gaussian where, through Bayesian techniques such as the E-M algorithm of Dempster, Laird, and Rubin (1977), the location parameters (such as the mean, μ_s) and dispersion or scale (such as the standard deviation σ_s) are estimated for each regime and a mixture law π is determined leading to a linear combination of the values of the two probabilities of each regime:

$$P(\mu_s, \sigma_s, \pi) = \pi \cdot \Phi(\mu_{s=1}, \sigma_{s=1}) + (1 - \pi) \cdot \Phi(\mu_{s=2}, \sigma_{s=2}) \tag{7}$$

Some of the applications of Gaussian mixture models in risk management or Financial Econometrics that can be mentioned are the works of Alexander and Lazar (2006), Bawuen, Hafner, and Rombouts (2007), Chung (2009), Haas, Mitnik, and Paolella (2004), Geweke and Amisano (2011), Nikolaev, Boshnakov, and Zimmer (2013), and Bezerra and Albuquerque (2017). For the specific case of the use of Gaussian mixture models in optimal portfolio selection "*à la Markowitz*," the works of Buckley, Sanders, and Seco (2008), Dark (2015), and Levy and Kaplanski (2015) can be cited. The authors concluded that parameter estimation with Gaussian mixtures in these three sources leads to more robust portfolio selection and better performance values. This is done by combining the two or S parameters corresponding to each regime or state of nature with the linear combination given in (**¡Error! No se**

[6] This state is also referred to as "crisis" in the related literature. This is not not to question or contravene, but rather to complement, other meanings of the term in the macroeconomic literature or in economic theory.

encuentra el origen de la referencia.). Nonetheless, as can be seen in (**¡Error! No se encuentra el origen de la referencia.**), it is possible to separate the behavior of the time series of the returns in S regimes, but the assumption is made that the possibility of being in a certain regime is fixed over time and is given by the value of the mixture law (π). Given this limitation, it is possible to resort to the MS or Markov-Switching models proposed by Hamilton (1989, 1994), reviewed in the introduction, and their extension of interest for the present study with the MS-GARCH models.

Given the dynamic nature of MS models and the possibility of inferring smoothed and transition probabilities for S regimes, many applications of these models have been studied. Examples of these in modeling crises in financial markets and their contagions to others can be found in the publications of Ang and Bekaert (2002b, 2002c), Kritzman, Page, and Turkington (2012), Klein (2013), Areal, Cortez, and Silva (2013), Zheng and Zuo (2013), and Hauptmann *et al*. (2014) (among others). These articles study the modeling of developed stock markets such as those of the United States, the United Kingdom, Germany, France, Switzerland, and Japan. These tests are performed in the presence of two or three volatility regimes.

Focusing their attention on other types of securities and their relation to other markets, the publications of Alexander and Kaeck (2007), Castellano and Scacia (2014), and Ma, Deng, and Ho (2018) can be mentioned. These papers study the behavior of credit default swaps (CDS) and the contagion of their performance in S regimes to other markets such as stock, foreign exchange, or oil markets.

Among the works that study applications in financial markets of emerging countries, the publications of Zhao (2010), Walid et al. (2011), Walid and Duc Khuong (2014), de Rotta and Valls-Pereira (2016), Mouratidis et al. (2013), Miles and Vijverberg (2011), Lopes and Nunes (2012), Kanas (2005), Álvarez-Plata and Schrooten (2006), Parikakis and Merika (2009), Girdzijauskas (2009), Dubinskas and Stungurienė (2010), Kutty (2010), Dufrénot, Mignon, and Péguin-Feissolle (2011), and Ahmed et al. (2018) can be mentioned. All of these publications focus on the study of S-regime modeling and the contagion of S-regimes among emerging stock and credit markets. They also review the influence of monetary policy on the change in the exchange rate regime or the stock markets.

Some of the most representative studies concerning the use of MS parameters (mean vectors and covariance matrices with regime switching) are seen in the works of Ang and Bekaert (2002a, 2004), Ishijima and Uchida (2011), and Kritzman, Page, and Turkington (2012), who suggest the inference of S covariance matrices and mean vectors. This is to perform the optimal selection according to Markowitz's optimal and rational selection proposals. Nevertheless, given the computational nature of this estimation and focusing the investment on a single type of asset, some alternatives can be considered.

Taking up the proposals of Tobin's Separation of Funds Theorem (1958), as well as the findings of Sharpe (1963, 1964), the investor's choice can be reduced to the selection of two types of assets: a risk-free one, which is usually the shortest-term money market instrument (such as a US Treasury bill with a 3-month maturity), as well as a risky asset, such as a theoretical portfolio that replicates the behavior of an index or market portfolio. Derived from this, one can reduce the n-dimensional problem (which involves optimally choosing the level of investment in n assets within the portfolio) to one that boils down to how much to invest in the risky asset and how much in the risk-free one (a one-dimensional problem). The same problem can be further simplified to a case in which it is decided to invest all the resources in the risky or risk-free asset if an algorithm or decision-making process makes it possible (such as the one proposed here).

Given this last idea and using the MS models, Brooks and Persand (2001) make a first attempt in which they determine whether to invest in the Gilt or 10-year bond of the United Kingdom or the FTSE100 stock index, given the probability that the $\text{FTSE100 gilt rate/dividend rate}$ ratio is in the high volatility regime or not. The results of these authors (which do not incorporate the impact of financing costs) suggest that employing this active management strategy leads to better performance results than a passive or "buy and hold" strategy in either the FTSE-100 or the British 10-year bond. Similarly, Hauptman *et al.* (2014) develop a warning system for the S&P500 index for a stochastic process with 3 regimes (low volatility, high volatility with an uptrend, and high volatility with a downtrend). The authors' method focuses on sequentially estimating the 3-regime model (to achieve a feasible result) and inferring the probabilities $\xi_{s=i,t}$ of being in a given regime by incorporating some exogenous factors. Their results (which also do not incorporate the impact of transaction costs) suggest that using an active strategy with MS models leads to better results than a passive or buy-and-hold strategy.

Finally, there is the work of De la Torre, Galeana, and Álvarez-García (2018). These authors explicitly study the application of MS models to investments in developed (United States, Italy, and the United Kingdom) and emerging (Mexico) stock markets. By incorporating the impact of a transaction cost of 0.35% plus value-added tax, the authors show that, in all three cases, using an active management strategy (the same as the one studied in this article) leads to better results than a passive one.

Given the above, it is observed, in the review of the literature as well as in the introduction, that previous works studying the use of MS models for investment decision-making focus only on the use of MS models with constant variance and practically all of them are focused on the application of these models in developed stock markets. Out of all these, only one focuses on an emerging market and none on studying the benefits of these strategies in frontier markets or markets in the Latin American region. As a result of this need, in terms of positive heuristics for financial economics and the use of MS and MS-GARCH models in investment decision-making, this paper will contribute to the literature, focusing on the particular cases of Mexico and Argentina. This is because, as previously mentioned, they are Latin American countries and one of them, Mexico, is considered emerging (with the most liquid currency of the emerging economies in the world), and the other is the third largest economy in the region and has been considered a frontier market in practically the entire simulated period. Given this, the comparison of these two countries, because of their characteristics and their regional location, is of interest to contribute to the published results in the use of MS models and MS-GARCH.

Since the need for this paper has been theoretically established, a brief contextualization of the use of the MS-GARCH model for investment decision-making purposes will now be given.

Methodology of the simulations performed

The MS-GARCH model and its use in active investment strategy

The MS-GARCH model to be used in the simulations of the present work is the one with the functional form given in (**¡Error! No se encuentra el origen de la referencia.**). Haas, Mitnik and Paolella (2004) note that the MS-GARCH model must be estimated once the residuals are determined from an arithmetic mean or some conditional mean model, such as (1), applied in r_t. Based on the fact that the present is one of the first applications of MS-GARCH models in frontier market investments, the assumption will be made that the appropriate model to determine the expected return or measure of location will be the arithmetic mean μ of r_t[7]. This will lead to $\varepsilon_t = r_t - \mu$. These residuals will be used to infer the MS, MS-ARCH, or MS-GARCH models, assuming that the index time series' generating or stochastic process has two homogeneous regimes (either Gaussian or t-Student distributed). Given this, the model will also permit the inference of the filtered probabilities and their corresponding smoothed probabilities ($\xi_{s,t}$) of being in the s regime at t.

The probability functions to obtain the filtered probabilities are given by the following expressions for the Gaussian and t-Student cases respectively[8]:

[7]The reason for using residuals as an estimation method is based on the theories of Haas, Mitnik, and Paolella (2004). This is because it is easier to estimate the model, given the time-dependent nature of the GARCH variances over time (which is interrupted by a regime change). Additionally, the mean will be used as a measure of central tendency since this is the first of future studies on the subject.

[8]For purposes of simplification in the review, homogeneous functions will be used in each regime, that is, the same in both estimated regimes. This is to achieve 6 simulation scenarios. Otherwise, there would be 12 scenarios and the development of the work would be more complex.

$$\xi_{s,i,t} = \frac{1}{\sqrt{2\pi}\sigma_{i,s}} e^{-\frac{1}{2}\left(\frac{\varepsilon_{i,t}}{\sigma_{i,s}}\right)^2}$$

(8)

$$\xi_{s,i,t} = \frac{\Gamma\left(\frac{\nu_{i,s}+1}{2}\right)}{\sqrt{(\nu_{i,s}-2)\pi}\Gamma\left(\frac{\nu_{i,s}}{2}\right)}\left(1+\frac{\left(\frac{\varepsilon_{i,t}}{\sigma_s}\right)^2}{(\nu_{i,s}-2)}\right)^{-\frac{\nu_{i,s}+1}{2}}$$

(9)

In the above expressions, ν_s represent the degrees of freedom of the t-Student distribution. These filtered likelihood functions will make it possible to infer the smoothed probabilities[9] and lead to the stable probabilities (or mixing laws) π_s of the next log-likelihood function to maximize[10]:

$$L(r_{i,t}, \theta) = \sum_{t}^{T} \ln\left(\sum_{S=1}^{S} \pi_s \cdot \xi_{s,i,t}\right), \theta = [\sigma_{i,s}, \pi_s, \mathbf{\Pi}]$$

(10)

To estimate the vector θ in (**¡Error! No se encuentra el origen de la referencia.**), a Bayesian maximum likelihood method will be used with the Viterbi algorithm (1967). Out of the parameters estimated with (**¡Error! No se encuentra el origen de la referencia.**) to (**¡Error! No se encuentra el origen de la referencia.**), particular attention was given to the smoothed probabilities $\xi_{s,t}$ at t, as well as to the transition probability matrix Π. With these parameters, the smoothed probability $\xi_{s,t+1}$of being in each regime in $t+1$ can be predicted. This as shown below:

$$\begin{bmatrix}\xi_{s=1,t+1} \\ \xi_{s=2,t+1}\end{bmatrix} = \mathbf{\Pi}\begin{bmatrix}\xi_{s=1,t} \\ \xi_{s=2,t}\end{bmatrix}$$

(11)

Given these two filtered probabilities, focusing now on $\xi_{s=2,t+1}$ and following what is established in the time series literature (Ang & Bekaert, 2002b; Brooks & Persand, 2001; Hamilton, 1989, 1990, 1994; Hauptmann *et al.*, 2014; Kritzman *et al.*, 2012), the investor can define whether they will be in a high or low volatility regime by using the following indicator function:

$$s_{t+1} = \begin{cases} 1 \ si \ \xi_{s=2,t+1} \leq 0.5 \\ 2 \ si \ \xi_{s=2,t+1} > 0.5 \end{cases}$$

[9]Estimated filtered probabilities according to the method of Kim (1994).

[10]Please refer to Hamilton (1989, 1994) for more detail on the development and rationale of the inference method.

(12)

Now that the MS-GARCH model and the parameters to be used in the investment strategy to be simulated have been explained, the following is a description of the pseudocode that governed the simulations performed.

The investment strategy pseudocode executed in the simulations

In order to perform the simulations that will test the usefulness of the investment decision-making process or investment strategy with MS-GARCH models, it will be assumed that the investor has a portfolio with an initial balance of USD 100 000.00 in which only two types of assets can be invested:

1. The base value 100, as of June 7, 2000, of the MSCI Argentina index — this index will be considered as the theoretical price of an exchange-traded fund (ETF) with a zero-tracking error toward the MSCI index of the simulated country.
2. The base value 100, also as of June 7, 2000, of a notional fund that pays the return rate on a 3-month US Treasury note with 3-month maturity (USTBILL) — this will be considered an investment fund with no tracking error.

Table 1 summarizes the stock indexes used as input data, the tickers (or identifiers used in this article) and the definition of the type of asset they were in the simulations.

Table 1
Stock indexes and risk-free assets of the countries to be used in the simulations

RIC® by Refinitiv™	Source of data	Index name	Ticker used in the article	Country	Type of asset in the simulations
.dMIAR00000PUS	Refinitiv™ Eikon™	MSCI ARGENTINA Index(USD)	MSCIARGUSD	Argentina	Risky asset (theoretical ETF)
.dMIMX00000PUS	Refinitiv™ Eikon™	MSCI Mexico Index (USD)	MSCIMEXUSD	Mexico	Risky asset (theoretical ETF)
UST3MT=RR	Refinitiv™ Eikon™	U.S. Treasury note with 3-month maturity.	USTBILL	United States	Risk-free assets (theoretical background)

Source: created by the author

The rationale for using the MSCI indices family is based on the fact that it is widely recognized and employed for benchmarking purposes in the investment policy design of internationally diversified portfolios (Bodie, Kane, & Marcus, 2014; Maggin et al., 2007).

The time series data of indices prices and the USTBILL rate were extracted from the Refinitiv™ Eikon™ databases (Refinitiv, 2018b) and transformed by the method of continuously capitalizable returns given in (**¡Error! No se encuentra el origen de la referencia.**). Subsequently, a time series of returns (r_t) with 1,078 observations (from January 1998) was determined. The three-time series began on June 6, 1998, and ended on January 31, 2019 (1,079 observations). As will be mentioned later, the simulations will start from January 1, 2000, through January 30, 2019 (996 simulation weeks) and the historical returns (r_t) for the previous weeks from June 1998 through t will be used for the MS-GARCH model parameter estimation.

Since this test is the first of several intended to be conducted for Latin American frontier and emerging stock markets, and in line with most of the previous related publications described in the literature review section, no brokerage costs or taxes will be considered in the purchase and sale transactions in the index. In addition, the impact of foreign exchange risk and market impacts due to price fluctuations when making purchase or sale transactions (slippage) will be left aside[11].

On the other hand, the simulated portfolio comprised only two sub-accounts:

1. A securities custody sub-account where the market value of the investments made in the assets previously described will be recorded, and
2. A cash subaccount in which the residual amounts that, given the number of bonds and the price of the invested assets, were left on demand, will be deposited

With these assumptions and parameters, the recursive execution of the following pseudocode will be simulated (in the 996 weeks of the simulated period):

Cycle, from week $t = 1$ to 996:

1. Quantify the current portfolio balance by adding the balance of the cash-on-demand subaccount plus the balance of the market value of the assets held in the custody subaccount
2. Run the analysis with the MS model (MS with constant variance, MS-ARCH or MS-GARCH) given in (**¡Error! No se encuentra el origen de la referencia.**). This is either with Gaussian likelihood function or homogeneous t-Student in the two regimes
3. Estimate the smoothed probability ($\xi_{s=2,t+1}$) of the high volatility regime for $t + 1$ with (**¡Error! No se encuentra el origen de la referencia.**)
4. If $s_{t+1} = 2$ ($\xi_{s=2,t+1} > 0.5$), then:

[11] As the indices of the MSCI family are originally calculated in U.S. dollars.

a. Invest in the fund that pays the USTBILL rate (Treasury-Bill ETF)

Otherwise:

b. Invest in the risky asset (the ETF replicating the simulated index's behavior)

5. Value the portfolio at mark-to-market prices with closing prices at t

End cycle

The results achieved for both the passive investment strategy (buy and hold) in the three simulated indices and the active management performed with the above pseudocode are presented in the following section.

Discussion of the simulation results

Statistical description of the data to be used in the simulations

The summary statistics of the time series of the historical returns of the stock indices and the weekly equivalent rate of the risk-free asset are shown in Table 2. As can be seen in the table, the average weekly average return paid by the MSCI Argentina is 0.20%, which is higher than the 0.0373% weekly return that the same investor would receive if they had invested their money in US Treasury notes[12]. Similarly, practically the same result can be seen for the Mexican case with an average return of 0.21%. One result that does show a differentiating factor between Argentina and Mexico is the range of minimum and maximum values. In these, the Argentinean index has more extreme magnitudes. This, of course, applies for the entire period under review from June 1998 to January 2019.

Table 2
Statistical summary of weekly returns of the stock indices and risk-free assets studied in this article (% values)

Ticker	Minimum	Quantile 5%	Mean	Standard deviation	Quantile 95%	Maximum
MSCIARGUSD	-28.5700	-8.1700	0.2000	5.4900	8.3100	30.3600
MSCIMEXUSD	-26.4200	-6.2000	0.2100	4.0100	5.8900	25.3000
USTBILL	-0.000254	0.000304	0.037354	0.039262	0.10538	0.12754

Source: created by the author with data from Thomson Reuters (2018b)

[12]Which is, in professional practice, considered a risk-free asset for an investor based on U.S. dollars.

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To determine whether the MS, MS-ARCH or MS-GARCH models are appropriate to approximate the stochastic process behavior of the time series of stock indices residuals, the Gaussian and t-Student likelihood function was calculated. This is considering the presence of a single regime. The Hamilton (1989, 1994) filter was then applied to infer the parameters of the MS, MS-ARCH and MS-GARCH models (Gaussian and t-Student[13]).

Table 3
Summary of Bayesian information criterion values for all Ms models studied for each stock index of interest

Stochastic process	MSCIARGUSD	MSCIMEXUSD
Gaussian (one regime)	-3 181.83	-3 858.55
t-Student (one regime)	-3 349.74	-4 038.77
MS-Gaussian	-3 364.2598[*]	-4 093.52
MS-tStudent	-3 359.65	-4 098.02
MSARCH-Gaussian	-3 350.29	-4 092.63
MSARCH-tStudent	-3 347.21	-4 093.82
MSARCH-Gaussian	-3 360.78	-4 108.14[*]
MSGARCH-tStudent	-3 354.12	-4 102.71

Source: created by the author with data from Thomson Reuters (2018b)

Once this was done, the Akaike (1974) information criterion was calculated. This was done because only one lag will be handled in both the ARCH and GARCH terms in the inferred model[14] and, since the number of parameters is fixed, there is no need to use a statistic that balances the number of parameters (model parsimony) and the accuracy of the estimated model. As there were a fixed number of parameters, in order to find the most accurate model to fit the time series, it was decided to use the AIC. With this in mind, Table 3 shows the comparative values of the AIC calculated for the time series of returns in both stock indices.

[13]Function that is homogeneous or the same for the two regimes.

[14]The MSGARCH library used in this work does not allow the calculation of ARCH and GARCH models with more than one lag in their terms. This is an extension that can be worked on in the future.

As can be seen, it is more appropriate to use a two-regime stochastic process to model the behavior of the time series of index returns (r_t). More precisely, among the 6 stochastic processes studied in this work, the Gaussian MS model with constant variance is the best for modeling the behavior of the Argentinean MSCI index while the Gaussian MS-GARCH is the best for the Mexican one.

With that explained, the simulations' results, where the six MS models proposed in each week were estimated, will now be reviewed.

Discussion and interpretation of the results observed in the simulations

As a first starting point, Table 4 presents the summary performance of the passive investment strategy (buy and hold) carried out either in the theoretical ETFs of the simulated indices or in the fund paying the USTBILL rate. As can be seen, and regardless of the average weekly return shown in Table 2, investing in the Argentinean stock market allowed the investor to achieve a cumulative return of 56.1372% (2.0040% per annum), which is higher than the 38.38% (2.9308 per annum) that investing their money in the USTBILLs would have paid. Similarly, the cumulative return of 162.06% (8.92% per annum) that the investor would have achieved on the MSCI Mexico can be seen.

For the period observed and given the existence of a financial and fiscal crisis in Argentina in the initial part of the study period, it can be seen that the MSCI Argentina had a higher cumulative and average return than the USTBILLs, but nearly a third of the cumulative return or almost half of the average return achieved in the Mexican case.

Table 4
Summary of passive management performance for each index and the risk-free asset under study (data in percentages except Sharpe ratio)

Ticker	Cumulative return	Average return	Return standard deviation	Max drawdown
MSCIARGUSD	56.1372 [2.9308]	0.0448	5.5518	-33.6472
MSCIMEXUSD	162.0699 [8.9213]	0.0968	3.8617	-30.6773
USTBILL	38.3854 [2.0040]	0.0374	0.0393	---

Source: calculated by the authors based on simulations using data from Thomson Reuters (2018b)

The risk exposure results are of additional interest — both the conditional Value at Risk (CVaR)[15] figures and the worst weekly performance (max drawdown[16]). The values for both stock indices are high and very close. Based on this result, evidence supports the hypothesis that, in addition to an improvement in the accumulated return, the level of risk exposure can be significantly reduced if active management is carried out with the investment system proposed herein.

Given this, the basic idea of active investment management is to reduce, on the one hand, these potential losses, as well as to increase the returns achieved through more periodic buying and selling operations. With this premise, it is expected to achieve these two objectives by executing the investment strategy suggested in this article, whose pseudocode was described in the previous section.

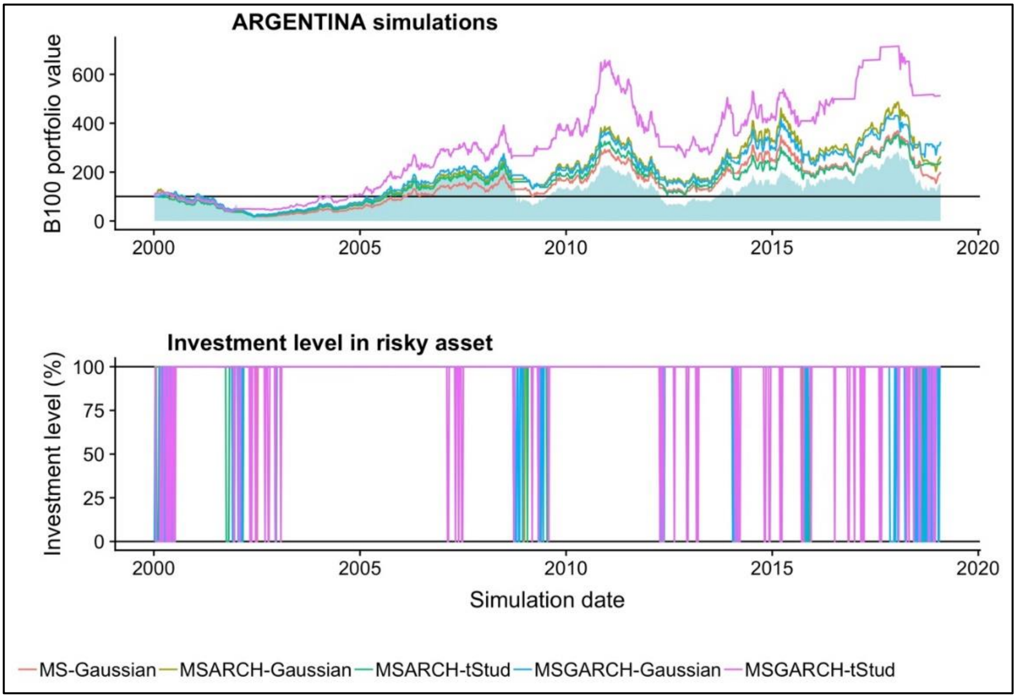

Figure 1. Historical performance of simulated portfolios in the Argentinean market
Source: calculated by the authors based on simulations using data from Thomson Reuters (2018b)

Table 5
Summary of active management performance, from the perspective of an investor with a US dollar-denominated portfolio, when investing in the MSCI Argentina index with Markov-Switching models

[15]Calculated using the following method: $CVaR = \int_{-\infty}^{\alpha} r_i p(r_i) dr_i,\ r_i \leq \alpha\,, \alpha = VaR(r_i)$ at 95% confidence.
[16]Calculated as $\min(r_{i,t})$.

Results observed in the Argentinean market					
Markov-Switching model used	Cumulative return	CVaR (98%)	Max drawdown	Sharpe ratio	Average level of investment in risky assets
MS-Gaussian	100.0754 [5.2248]	-12.0474	-14.9487	-0.0041	96.15
MS-tStudent	Not feasible	Not feasible	Not feasible	Not feasible	Not feasible
MSARCH-Gaussian	164.3827 [8.5822]	-12.0135	-14.9517	0.0017	95.15
MSARCH-tStudent	143.581 [7.4962]	-12.4817	-19.4665	-0.0049	95.53
MSARCH-Gaussian	223.8074 [11.6847]	-11.9985	-14.9551	0.0082	92.74
MSGARCH-tStudent	412.8318 [21.5535]	-11.5586	-19.4741	0.0162	92.05

Results observed in the Mexican market					
Markov-Switching model used	Cumulative return	CVaR (98%)	Max drawdown	Sharpe ratio	Average level of investment in risky assets
MS-Gaussian	259.3627 [13.541]	-8.3678	-17.4144	0.0147	0.9676
MS-tStudent	255.8699 [13.3587]	-7.9479	-10.1805	0.0223	0.9636
MSARCH-Gaussian	75.7788 [3.9563]	-7.8676	-10.1763	-0.0048	0.9493
MSARCH-tStudent	95.3431 [4.9778]	-8.8926	-17.4112	0.0034	0.9713
MSARCH-Gaussian	154.7175 [8.0776]	-7.8395	-10.1853	0.011	0.9172
MSGARCH-tStudent	37.3705 [1.9511]	-8.8326	-14.0064	-0.0054	0.9351

Source: Created by the authors with data from simulations performed and Thomson Reuters (2018b)

To verify this and after performing the event simulations previously described, the results observed for the Argentinean case are presented. This is shown in the upper part of Table 5 (the values of all fields, except Sharpe ratio, are presented in percentages). As can be seen, using MS-GARCH t-Student models leads to the best performance, with a cumulative return of 412.8318% (21.5535% per annum). It

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is followed by the Gaussian MS-GARCH model with 223.8074% (11.6847%), the Gaussian MS-ARCH with 164.3827% (8.5822%), the distributed MS-ARCH t-Student with 143.5810% (7.4962%), and the Gaussian MS (with constant variance) with 100.0754% (5.2248%) return. For the specific case of the MS model with constant variance and t-Student likelihood function, the legend "not feasible" is observed. This implies that, in some of the simulation weeks[17], the estimation method did not lead to convergence in the maximum likelihood problem employed and, therefore, no values for the probability of being in the high volatility period at $t + 1$ were available. Given this, the use of this model was excluded from the simulation.

As a summary of the accumulated return results, it can be seen that, for the Argentinean case, the objective of this work is achieved, and the hypothesis states that an additional return is generated to the passive strategy (Alpha) if the investment strategy proposed herein is used.

Additionally, the improvement in the level of risk exposure in the simulated portfolios can be seen. For example, the worst weekly performance (max drawdown) observed was the case of the portfolio with MS-GARCH t-Student models, which had a drawdown of -19.7441%. This is notably lower than the worst weekly performance observed in the passive or buy-and-hold strategy (Table 4), which presented a drop of -33.6472%. With this, the second hypothesis of a reduction in the level of risk exposure is tested for the Argentinean case.

To provide a better basis for presenting results, Figure 1 shows the historical performance of the six simulated portfolios in this market (Argentina). It shows that the MS-GARCH t-Student model scenario has superior results to a "buy and hold" strategy (its performance is shown as a shaded area). The observed result is attributable to the fact that the use of the Gaussian MS-GARCH model led to more accurate investment decisions. This can be seen in well-known periods of high volatility or crisis in the markets, such as October 2008, the European debt crisis of March-July 2013, July-November 2016, and the last months of 2019. These periods are known for being episodes of crisis in the financial markets or political situations that significantly increased volatility levels. This was to the extent that they were identified as periods of the second regime, leading to a decision to disinvest in the stock market. These decisions to invest in the risk-free asset allowed the portfolio simulated in this scenario to behave in a relatively flat manner during the fall of the markets in that period, since it was not exposed to the level of risk observed at that time. The simulated portfolio achieved better performance results because it was

[17] The dates on which convergence was not achieved in the optimization problem were the following 6: February 2, 2001, September 14, 2001, April 21, 2006, October 6, 2006, November 3, 2006, and February 2, 2007. The reasons for this stem from issues specific to the method of inference used. Given this, it is suggested to use alternative methods such as Markov Chain Monte Carlo simulation.

disinvested in periods of generally declining stock prices. The rationale for using these models (MS-GARCH t-Student) for investment decisions in the Argentinean market can be observed.

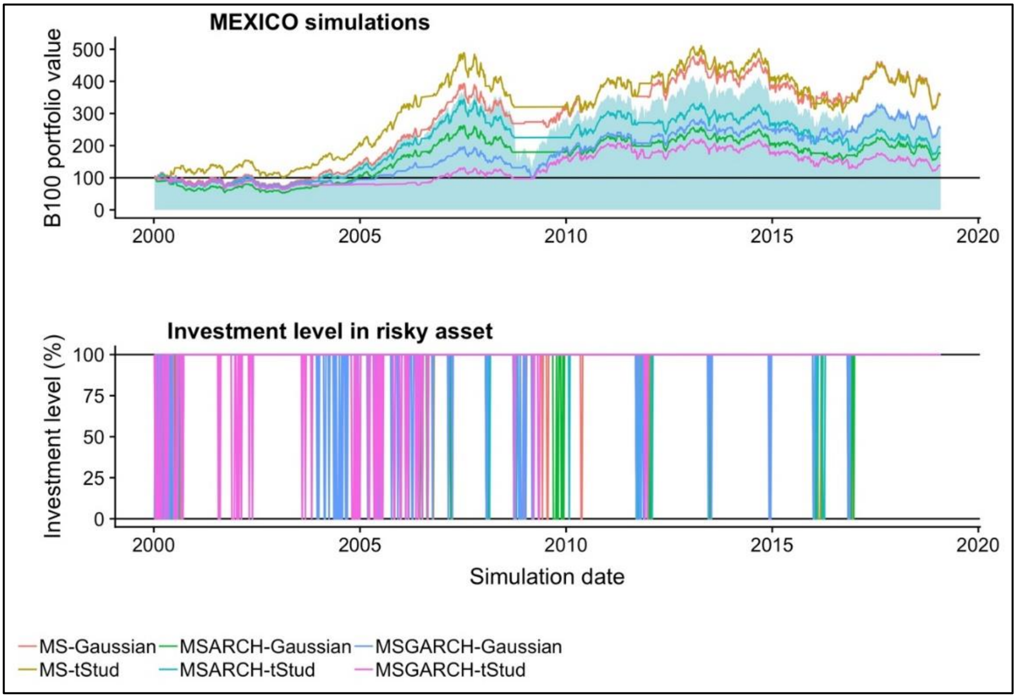

Figure 2. Historical performance of simulated portfolios in the Mexican market
Source: calculated by the authors based on simulations using data from Thomson Reuters (2018b)

For the Mexican case, the results can be seen in the lower panel of Table 5. As can be seen, the Gaussian MS model with constant variance is the one that leads to the best cumulative return results, with a 259.36% cumulative return (13.51% annualized), followed by the same model with t-Student likelihood function that achieved 255.86% (13.35%). As can be seen, in this particular case, *Alpha* or additional returns to those achieved with a passive strategy are also achieved. Nevertheless, this is achieved with constant variance MS models. This result is of interest as it contradicts both the goodness of fit results in Table 3 and those observed in the Argentinean market, where it is better to use MS-GARCH models. This leads to a result of potential interest and future research guidance. It also highlights that in Mexico, an emerging Latin American market, MS models with constant variance are more appropriate for active management, and in a "frontier" market such as Argentina, the MS-GARCH model is more attractive.

Concerning the levels of risk exposure observed for the simulated portfolio in this market, it can be seen that, as in the Argentinean case, the second hypothesis is fulfilled, which states that there is an observable reduction in risk exposure if active portfolio management is carried out.

Similarly to the analysis carried out in the Argentinean market, Figure 2 is presented. The historical performance of the simulated portfolios in this country is presented and it can be seen why the scenario using the distributed MS t-Student model has the best performance results. Specifically, it can be seen how in the period from October 2008 to February 2009, this portfolio was disinvested in the stock market. This result can be seen in its behavior, which was a straight line. The latter is the result of being invested in a risk-free asset. Furthermore, the portfolio had zero investment levels in the stock market during this period and in the crisis period in mid-2013. This made it possible to generate important differences in terms of performance, concerning a “buy and hold” strategy (shaded area in the graph).

As a corollary of the observed results, it can be noted that using MS-GARCH models leads to better performance results and appropriate investment decisions. The latter allows for improved performance concerning a passive investment strategy. Additionally, and as a basis for the above, the decision-making achieved with these models reduces the level of risk exposure. Specifically, in times of very high volatility, the model reduces equity positions so that downward movements are not affected.

The results show that using MS-GARCH models with the Gaussian likelihood function (for the Argentinean case) and the MS t-Student (with fixed variance over time) lead to the best performance. Perhaps, for the Mexican case, the working hypothesis was not completely fulfilled (the best performance is not achieved with an MS-GARCH model, but with an MS model), but the relevance of using this model for decision-making was proven. In addition, using MS-GARCH models permitted more appropriate decisions in a frontier market such as the Argentinean one. This could lead to the assumption that the use of MS models with constant variance is appropriate in markets that are not categorized as frontier and that the use of MS-GARCH models is appropriate for frontier markets or markets with a higher level of risk exposure.

Conclusions

Markov-Switching or MS (Hamilton, 1989, 1990, 1994) models were proposed to model time series. This applies when their behavior is typical of a stochastic process with $s \geq 2$ regimes or states of nature. That is, it must be a stochastic process coming not from one but from two or more probability functions, which have $s \geq 2$ parameters of location, dispersion, and shape.

At the time of writing, multiple applications of this model have been proposed and studied. Examples of these are quantifying the probability that a given economy is in a recession. The contagion effect (spillovers) of a central bank's monetary policy on the exchange or stock markets has also been studied.

Another little-studied application of MS models is their use in the decision-making process in active investment strategies. Specific cases of this application are the classic works of Brooks and Persand (2001), Ang and Bekaert (2002a), Kritzman, Page, and Turkington (2012), and Hauptmann et al. (2014). Others are the tests conducted by De la Torre, Galeana, and Alvarez-Garcia (2018) that focus on using MS models in the active management of stock indices of developed countries and Mexico (an emerging country). Based on this review, little has been studied in detail on the subject (use of MS models in investments) for other emerging economies, and nothing has been written on the use of MS, MS-ARCH, or MS-GARCH models in developed, emerging, and frontier markets. Given this lack, this analysis is carried out in Argentina, considered a frontier stock market, and Mexico, an emerging country, from the perspective of an investor whose portfolio is denominated in US dollars.

In addition, it was observed that the papers that test active management models with MS models only do so with a Gaussian likelihood function and with conventional or constant variance over time. Given this, the present study extends the subject by using MS models with ARCH or GARCH variances and contrasts the performance results that would be achieved if an investor had actively managed portfolios with them. This is achieved by using the following investment strategy:

1. Invest in the market index of the country under study if the investor expects to be in the normal or low volatility regime ($s = 1$) in $\mathrm{t} + 1$ or
2. Invest in the US risk-free asset if the investor expects, for the same period ($\mathrm{t} + 1$), to be in the ($s = 2$) high volatility regime.

With this investment strategy, the demonstration of two working hypotheses that are complementary to each other was proposed:

H_1: "The use of MS-GARCH models in the active investment strategy generates, in the Argentinean and Mexican stock markets, Alpha or additional returns compared to a passive strategy of the buy and hold type."

H_2: "The use of MS-GARCH models in the active investment strategy generates a significant reduction in risk exposure in the Argentinean and Mexican stock markets. This is compared to a passive buy-and-hold strategy."

To test these two hypotheses, weekly simulations were performed from January 2000 to January 2019 (996 weeks) in the Argentinean and Mexican stock markets. The MS, MS-ARCH, or MS-GARCH models were also inferred recursively at each date. Given this, the smoothed probabilities of being in the high volatility regime at $\mathrm{t} + 1$ were estimated to realize the investment strategy previously described. The

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results of the simulations showed that the MS-GARCH models with t-Student likelihood function lead, for the case of the Argentinean market, to cumulative returns of 412.8318% (21.5535% per annum), compared to a cumulative return of 56.1372% (2.9308% p.a.) and 38.3854% (2.0040% p.a.) achieved by passively investing in the MSCI Argentina index and US Treasury bills. In addition, it can be seen that the levels of risk exposure are reduced thanks to this active management.

For the specific case of the Mexican stock market, it is observed that the use of the MS Gaussian models with constant variance leads to a 259.36% cumulative return (13.51% annualized), a value that is notably higher than the 162.06% (8.92% annualized) observed if the passive strategy is used in this market.

As can be seen when contrasting an emerging Latin American market (Mexico) and a frontier market in the same region (Argentina), the working hypotheses proposed in this article are fully fulfilled for the Argentinean market. For the Mexican case, the first working hypothesis is partially fulfilled. Alpha is generated, not through MS-GARCH models, but through MS Gaussian models with constant variance, the original proposal of Hamilton (1989, 1994).

The observed results have implications of potential interest for the professional practice of investment management, since the use of MS models with constant variance and MS-GARCH enable the generation of Alpha or marginal returns for a passive strategy and would permit a dollar-denominated investor to buy and sell the MSCI Argentina or the MSCI Mexico. This is according to the expectations or probability of being in a high volatility regime at $t + 1$. This practice may revolutionize the investment companies market or the exchange-traded funds (ETFs) market on both the supply and demand side. This is because the selection of two assets, proposed in this investment strategy in Mexico (emerging market) and Argentina (frontier market), would enable the investor to actively invest in these types of markets more precisely (in terms of making investment and divestment decisions) and through a single diversified vehicle (the ETF or investment company).

Among the areas of opportunity identified for future research work and given the limitations of the work indicated in the review of the results, the following guidelines are proposed:

1. Develop and simulate investment strategies with more than two regimes.
2. Simulate these investment strategies with MS, MS-ARCH, or MS-GARCH models with heterogeneous likelihood functions in each regime.
3. Use asymmetric models in volatility parameters and likelihood functions.
4. Incorporate the impact of financial transaction costs, as well as other market risks not incorporated here, such as slippage (or fluctuation of the execution price), foreign exchange rate risk, or any other risk or impact due to the influence of exogenous variables or events not incorporated in the MS model used.

The results presented here are expected to significantly contribute to the literature on the use of MS models—specifically, to their use for active portfolio management purposes in securities markets classified as "frontier markets" and in "emerging markets." Additionally, it is expected to contribute to the study of the benefits of active investment using MS-GARCH models.

References

Ahmed, R. R., Vveinhardt, J., Štreimikiene, D., Ghauri, S. P., y Ashraf, M. (2018). Stock returns, volatility and mean reversion in Emerging and Developed financial markets. Technological and Economic Development of Economy, 24 (3): 1149-1177.

Akaike, H. (1974). A new look at the statistical model identification. IEEE transactions on automatic control, 19 (6): 716-723.

Alexander, C., y Kaeck, A. (2007). Regime dependent determinants of credit default swap spreads. Journal of Banking & Finance, (32): 1008-1021.

Alexander, C., y Lazar, E. (2006). Normal mixture GARCH(1,1): Applications to exchange rate modelling. Journal of Applied Econometrics, 21 (3): 307-336.

Alvarez-Plata, P., y Schrooten, M. (2006). The Argentinean currency crisis: A Markov-switching model estimation. Developing Economies, 44 (1): 79-91.

Ang, A., y Bekaert, G. (2002a). International Asset Allocation With Regime Shifts. The review of financial studies, 15 (4): 1137-1187.

Ang, A., y Bekaert, G. (2002b). Regime Switches in Interest Rates. Journal of Business & Economic Statistics, 20 (2): 163-182.

Ang, A., y Bekaert, G. (2002c). Short rate nonlinearities and regime switches. Journal of Economic Dynamics and Control, 26 (7-8): 1243-1274.

Ang, A., y Bekaert, G. (2004). How regimes affect asset allocation. Financial Analysts Journal, 60 (2): 86-99.

Areal, N., Cortez, M. C., y Silva, F. (2013). The conditional performance of US mutual funds over different market regimes: do different types of ethical screens matter? Financial Markets and Portfolio Management, 27 (4): 397-429.

Bank for International Settlements. (2016). Triennial Central Bank Survey of foreign exchange and OTC derivatives markets in 2016. Recuperado at 12 de diciembre de 2018, a partir de: http://www.bis.org/publ/rpfx16. htm?m=6%7C35

Bauwen, L., Hafner, C. M., y Rombouts, J. V. K. (2007). Multivariate mixed normal conditional heteroskedasticity. Computational Statistics & Data analysis, 51 (7): 3551-3566.

Bezerra, P. C. S., y Albuquerque, P. H. M. (2017). Volatility forecasting via SVR–GARCH with mixture of Gaussian kernels. Computational Management Science, 14 (2): 179-196.

Black, F., y Litterman, R. (1992). Global portfolio optimization. Financial Analysts Journal, 48 (5): 28-43.

Bodie, Z., Kane, A., y Marcus, A. (2014). Investments global edition (10th ed.). New York, USA: Mc Graw-Hill.

Bollerslev, T. (1987). A Conditionally Heteroskedastic time series model for speculative prices and rates of return. The Review of Economics and Statistics, 69 (3): 542-547.

Brooks, C., y Persand, G. (2001). The trading profitability of forecasts of the gilt–equity yield ratio. International journal of forecasting, 17 (1): 11-29.

Buckley, I., Saunders, D., y Seco, L. (2008). Portfolio optimization when asset returns have the Gaussian mixture distribution. European Journal of Operational Research, 185 (3): 1434-1461.

Cabrera, G., Coronado, S., Rojas, O., y Venegas-Martínez, F. (2017). Synchronization and Changes in Volatilities in the Latin American'S Stock Exchange Markets. International Journal of Pure and Applied Mathematics, 114 (1).

Camacho, M., y Perez-Quiros, G. (2014). Commodity Prices and the Business Cycle in Latin America: Living and Dying by Commodities? Emerging Markets Finance and Trade, 50 (2): 110-137.

Castellano, R., y Scaccia, L. (2014). Can CDS indexes signal future turmoils in the stock market? A Markov switching perspective. CEJOR, 22 (2): 285-305.

Chung, S. K. (2009). Bivariate mixed normal GARCH models and out-of-sample hedge performances. Finance Research Letters, 6 (3): 130-137.

Dark, J. (2015). Futures hedging with Markov switching vector error correction FIEGARCH and FIAPARCH. Journal of Banking and Finance, 61 : S269-S285.

De la Torre, O., Galeana-figueroa, E., y Álvarez-García, J. (2018). Using Markov-Switching models in Italian , British , U . S . and Mexican equity portfolios : a performance test. Electronic Journal of Applied Statistical Analysis, 11 (2): 489-505.

Dempster, A. P., Laird, N. M., y Rubin, D. B. (1977). Maximum Likelihood from Incomplete Data via the EM Algorithm. Journal of the Royal Statistical Society. Series B (Methodological), 39 : 1-38.

Dubinskas, P., y Stungurienė, S. (2010). Alterations in the financial markets of the baltic countries and Russia in the period of Economic cowntum. Technological and Economic Development of Economy, 16 (3): 502-515.

Dueker, M. (1997). Markov Switching in GARCH Processes and Mean- Reverting Stock-Market Volatility. Journal of business & Economics Statistics, 15 (1): 26-34.

Dufrénot, G., Mignon, V., y Péguin-Feissolle, A. (2011). The effects of the subprime crisis on the Latin American financial markets: An empirical assessment. Economic Modelling, 28 (5): 2342-2357.

Engle, R. (1982). Autoregressive Conditional Heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50 (4): 987-1007.

Fama, E. (1965). The behavior of stock-market prices. Journal of business, 38 (1): 34-105.

Fama, E. F. (1963). Mandelbrot and the Stable Paretian Hypothesis. The journal of business, 36 (4): 420-429.

Geweke, J., y Amisano, G. (2011). Hierarchical Markov normal mixture models with applications to financial asset returns. Journal of Applied Econometrics, 26 (1): 1-29.

Girdzijauskas, S., Štreimikienė, D., Čepinskis, J., Moskaliova, V., Jurkonytė, E., y Mackevičius, R. (2009). Formation of Economic bubles: cuases and possible interventions. Technological and Economic Development of Economy, 15 (2): 267-280.

Glosten, L., Jaganathan, R., y Runkle, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. The Journal of Finance, 48 (5): 1779-1801.

Haas, M, Mittnik, S., y Paolella, M. S. (2004). Mixed normal conditional heteroskedasticity. Journal of financial Econometrics, 2 (2): 211-250.

Haas, Markus, Mittnik, S., y Paolella, M. S. (2004). A New Approach to Markov-Switching GARCH Models. Journal of financial Econometrics, 2 (4): 493-530.

Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. Econometrica, 57 (2): 357-384.

Hamilton, J. D. (1990). Analysis of time series subject to changes in regime. Journal of Econometrics, 45 (1-2): 39-70.

Hamilton, J. D. (1994). Time Series Analysis. Princeton: Princeton university press.

Hamilton, J. D., y Susmel, R. (1994). Autoregressive conditional heteroskedasticity and changes in regime. Journal of Econometrics, 64 (1-2): 307-333.

Hauptmann, J., Hoppenkamps, A., Min, A., Ramsauer, F., y Zagst, R. (2014). Forecasting market turbulence using regime-switching models. Financial Markets and Portfolio Management, 28 (2): 139-164.

Ishijima, H., y Uchida, M. (2011). The Regime Switching Portfolios. Asia-Pacific Financial Markets, 18 (2): 167- 189.

Jorion, P. (1992). Portfolio optimization in practice. Financial analysts journal, 48 (1): 68-74.

Kanas, A. (2005). Regime linkages between the Mexican currency market and emerging equity markets. Economic Modelling, 22 (1): 109-125.

Kim, C.-J. (1994). Dynamic linear models with Markov-switching. Journal of Econometrics, 60 (1-2): 1-22.

Klaassen, F. (2002). Improving GARCH volatility forecasts with regime-switching GARCH. En Advances in Markov-Switching Models (223-254). Heidelberg: Physica-Verlag HD.

Klein, A. C. (2013). Time-variations in herding behavior: Evidence from a Markov switching SUR model. Journal of International Financial Markets, Institutions & Money, 26 : 291-304.

Kritzman, M., Page, S., y Turkington, D. (2012). Regime Shifts: Implications for Dynamic Strategies. Financial Analysts Journal, 68 (3): 22-39.

Kutty, G. (2010). the Relationship Between Exchange Rates and Stock Prices : the Case of Mexico. North American Journal of Finance and Banking Research, 4 (4): 1-12.

Lamoureux, C. G., y Lastrapes, W. D. (1990). Persistence in Variance, Structural Change, and the GARCH Model. Journal of Business & Economic Statistics, 8 (2): 225-234.

Levy, M., y Kaplanski, G. (2015). Portfolio selection in a two-regimeworld. European Journal of Operational Research, 242 (2): 514-524.

Lopes, J. M., y Nunes, L. C. (2012). A Markov regime switching model of crises and contagion: The case of the Iberian countries in the EMS. Journal of Macroeconomics, 34 : 1141-1153.

Ma, J., Deng, X., Ho, K.-C., y Tsai, S.-B. (2018). Regime-Switching Determinants for Spreads of Emerging Markets Sovereign Credit Default Swaps. Sustainability, 10 (2730): 1-17.

Maggin, J. L., Tuttle, D., Pinto, J., y McLeavey, D. W. (2007). Managing Investment Portfolios: A Dynamic Process. (John Miley and Sons Inc, Ed.). Hoboken, USA.

Markowitz, H. (1959). Portfolio selection. Efficient diversification of investments. New Haven: Yale University Press.

Markowitz, Harry. (1952). Portfolio selection. The Journal of Finance, 7 (1): 77-91. Markowitz, Harry. (1956). The optimization of quadratic functions subject to linear constraints. Naval research logistic quarterly, 3 (March-June): 1-113.

Michaud, R. (1989). The Markowitz Optimization Enigma: Is optimized optimal? Financial Analysts Journal, 45 (1): 31-42.

Michaud, R., y Michaud, R. (2008). Efficient asset management. A practical guide to stock portfolio optimization and asset allocation. New York: Oxford university press.

Miles, W., y Vijverberg, C.-P. (2011). Formal targets, central bank independence and inflation dynamics in the UK: A Markov-Switching approach. Journal of Macroeconomics, 33 : 644-655.

Mouratidis, K., Kenourgios, D., Samitas, A., y Vougas, D. (2013). Evaluating currency crises: A multivariate markov regime switching approach*. Manchester School, 81 (1): 33-57.

MSCI Inc. (2018). MSCI Global Investable Market Indexes Methodology. Recuperado at 2 de mayo de 2018, a partir de: http://www.msci.com/eqb/methodology/meth_docs/MSCI_Jan2015_GIMIMethodology_vf.pdf

Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica, 59 (2): 347.

Nikolaev, N. Y., Boshnakov, G. N., y Zimmer, R. (2013). Heavy-tailed mixture GARCH volatility modeling and Value-at-Risk estimation. Expert Systems with Applications, 40 (6): 2233-2243.

Parikakis, G. S., y Merika, A. (2009). Evaluating volatility dynamics and the forecasting ability of Markov switching models. Journal of Forecasting, 28 (8): 736-744.

Refinitiv. (2018a). Lipper Fund Research | Refinitiv. Recuperado at 27 de noviembre de 2019, a partir de: https:// www.refinitiv.com/en/products/lipper-fund-research/?utm_content=Product Name-LATAM-AMER-GENExact&utm_medium=cpc&utm_source=google&utm_campaign=68832_RefinitivBAUPaidSearch&elqCampaignId=5917&utm_term=lipper mutual fund&&gclid=Cj0KCQiA2vjuB

Refinitiv. (2018b). Refinitiv Eikon. Recuperado at 3 de junio de 2019, a partir de: https://eikon.thomsonreuters. com/index.html

Rotta, P. N., y Valls Pereira, P. L. (2016). Analysis of contagion from the dynamic conditional correlation model with Markov Regime switching. Applied Economics, 48 (25): 2367-2382.

Sharpe, W. (1963). A simplified model for portfolio analysis. Management Science, 9 (2): 277-293.

Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. The journal of finance, XIX (3): 425-442.

Sosa, M., Ortiz, E., y Cabello, A. (2018). Dynamic Linkages between Stock Market and Exchange Rate in mila Countries: A Markov Regime Switching Approach (2003-2016). Análisis Económico, 33 (83): 57-74.

Tobin, J. (1958). Liquidity preference as behavior toward risk. The Review of Economic Studies, XXV (1): 65-86.

Viterbi, A. (1967). Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. IEEE Transactions on Information Theory, 13 (2): 260-269.

Walid, C., Chaker, A., Masood, O., y Fry, J. (2011). Stock market volatility and exchange rates in emerging countries: A Markov-state switching approach. Emerging Markets Review, 12 : 272-292.

Walid, C., y Duc Khuong, D. (2014). Exchange rate movements and stock market returns in a regime-switching environment: Evidence for BRICS countries. Research in International Business and Finance, (31): 46-56.

World Bank. (2019). GDP (current US$). Recuperado at 3 de julio de 2019, a partir de: https://data.worldbank.org/ indicator/NY.GDP.MKTP.CD?locations=IN&view=map

Xiao, Y., y Valdez, E. a. (2015). A Black–Litterman asset allocation model under Elliptical distributions. Quantitative Finance, 15 (3)

Zhao, H. (2010). Dynamic relationship between exchange rate and stock price: Evidence from China. Research in International Business and Finance, 24 (2): 103-112.

Zheng, T., y Zuo, H. (2013). Reexamining the time-varying volatility spillover effects: A Markov switching causality approach. North American Journal of Economics and Finance, 26 : 643-662.