



Optimization of VAT payment through a probabilistic risk model for financial planning

Optimización del pago del IVA a través de un modelo de riesgo probabilístico para la planeación financiera

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Abstract

The Small and Medium Sized Enterprises (SMEs) have an important fiscal responsibility, so it is of special interest to reach an appropriate management of the cash flow, in order to optimize the payment of the Value Added Tax. In this work, a cost-risk analysis model is proposed to determine the impact of a fiscal planning strategy that considers the use of a credit line. The goal is to make a decision that achieves an optimal combination between costs and expected benefits, considering also the risk associated with the occurrence of lack of liquidity. This proposal is focused on the minimization of what is called the total impact function, defined as the sum of the cost and risk functions. The methodological proposal is illustrated with real data from a hardware wholesale company in Hermosillo, Sonora, Mexico. A simulation study was carried out to show the applicability of this proposal.

JEL Code: C51, C61, M10, M19

Keywords: model construction and estimation; optimization techniques; programming models; dynamics analysis; general

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Resumen

Las pequeñas y medianas empresas (PYMES) tienen una responsabilidad fiscal muy importante, por lo que resulta de especial interés administrar el flujo de efectivo para optimizar el pago del Impuesto al Valor Agregado. En este trabajo se propone un modelo de análisis de costo-riesgo para determinar el impacto de una estrategia de planeación fiscal que considera la intervención de una línea de crédito. Se busca tomar una decisión que logre una combinación óptima entre costos y beneficios esperados, considerando el riesgo asociado a la ocurrencia de falta de liquidez. La propuesta considera minimizar la función de impacto total, la cual se define como la suma de la función de costo y la función de riesgo. La propuesta metodológica se ilustra con datos reales de una empresa mayorista ferretera en Hermosillo, Sonora, México. Además, se llevó a cabo un estudio de simulación para mostrar la pertinencia de la propuesta.

Código JEL: C51, C61, M10, M19

Palabras clave: construcción de modelos y estimación; técnicas de optimización; modelos de programación; sistema dinámico; generalidades

Introduction

Companies define their objectives and include their tax strategies considering legal aspects and yield maximization after paying taxes, such as income tax, indirect taxes, and employee and property taxes. Ideally, these strategies should seek to reduce the tax burden to enable them to allocate more resources to operations and brand positioning, as well as to pay dividends to shareholders or to reinvest profits.

Currently, most strategies are based on reducing the taxable base for income tax purposes, but in this case, the objective is to reduce the burden of an indirect tax, such as the Value Added Tax (VAT). Proper planning can optimize the tax burden to minimize its impact on cash flow and obtain operational and financial improvements.

This paper presents a VAT strategy, which will serve as a planning model since the objective will be to reduce the tax payable, resulting from the difference between the tax collected and the creditable tax, by increasing the creditable tax through a strategy of obtaining a line of credit, of which all withdrawals will be exclusively for making payments to suppliers. A model is developed to determine the optimal level of credit for this purpose, and the associated risks are modeled.

To the authors' knowledge, no work of this kind has been done since the analyses on the subject have only focused on the accounting aspect of the problem, seeking to reduce the tax payable without considering other aspects included in this paper, such as the risk model, to give more realism to the business scenario in which they are applied.

Theoretical framework

A value-added tax (VAT) is a consumption tax applied to a product repeatedly at each point of sale at which value has been added. That is, the tax is added when a producer of raw materials sells a product to a factory, when the factory sells the finished product to a wholesaler, when the wholesaler sells it to a retailer, and finally, when the retailer sells it to the consumer who will use it. Ultimately, the consumer pays the VAT. The down-the-chain buyer refunds the VAT to each earlier-stage buyer of the product's production. When the company sells a product, it collects the corresponding tax percentage, whereas when the company buys inputs or contracts services, it pays the VAT percentage to its suppliers. The difference between the VAT collected and the VAT paid becomes the VAT payable by the company to the government. VAT affects the company's operating cash flow. If the VAT collected is greater than the VAT paid, a liability is generated for the company. While VAT is collected, it generates an inflow of cash flow for the company; however, if it is paid, this cash flow decreases. To the extent that cash flow affects operating income, it is concluded that proper VAT management can create value for the company or otherwise negatively affect its present value. Likewise, to the extent that the VAT collected is a liability for the company, it affects its short-term financial position and impacts liquidity risk, such that inadequate VAT management can increase this risk or, on the contrary, decrease it.

Baena (2014) explains that financial analysis is a procedure by which qualitative and quantitative data are analyzed and compared with historical economic events and current events occurring in the entity. Its purpose is to identify the company's real situation and take the necessary measures to correct possible management errors. This process is particularly relevant in the context of small companies characterized by credit astringency, as Nason and Nordqvist (2020) pointed out.

The problem in question is located in the management of the cash conversion cycle, specifically in the operating cycle (see Zutter & Smart, 2019), where the aim is to make the inventory management phase more efficient, optimizing the inventory cost. This is so since, through the line of credit, substantial discounts are obtained on purchases and a reduction in the tax cost due to the effect of deducting the financial expenses derived from the credit. The financing proposal is neutral with respect to the average payment period since the tax liability is transferred to short-term bank debt.

This paper takes as a reference, among others, the work of Lemieux (2013), which seeks to visually reflect the process of optimal financial decision making and risk management. Similarly, the work of Myznikova and Zhdanova (2017) suggests the development of cash flow optimization models derived from the company's own cash flows. Kádárová et al. (2015) agree that the optimal management of operating flows can define the future and survival of a company. The proposal starts with the recognition

of fiscal obligations (Fishman, 2020). Of course, these activities are also placed in the context of optimal cash flow management; see Reider and Heyler (2003).

In particular, this paper proposes a cost-risk analysis model to determine the impact of a fiscal planning strategy that considers the intervention of a line of credit. Such a model is proposed since most business decisions involve an aspect of cost-risk trade-off. In general, cost is a formula used to predict the cost experienced at a given activity level. On the other hand, the definition of risk—developed under the umbrella of the United Nations (UNDRO, 1979)—includes the following elements: the probability of occurrence of a potentially hazardous event (hazard); the degree of loss resulting from the probable occurrence of the hazardous event (vulnerability); and the elements at risk (exposure). For example, some works that illustrate the definition of risk in the construction of models for risk analysis in ecosystems, floods, and urban planning are Van Oijen et al. (2013), Špačková and Straub (2015), and Maragno et al. (2020), respectively.

Traditionally, cost and risk analysis procedures have been proposed for engineering, maintenance, and operation activities, taking the cost-risk optimization model as a theoretical framework. Under this approach, a decision is sought to achieve an optimal combination between costs and expected benefits, considering the risk associated with the occurrence—or non-occurrence—of a hazardous event or threat. In particular, the cost-risk optimization model allows decisions based on the concept of minimum total impact. In this context, the total impact is a function defined as the sum of the cost and risk functions. For further details, see Woodhouse (1993).

Materials and methods

This section describes the components and procedures involved in the process of optimizing the operation of a company based on the minimization of a total impact function that considers cost, risk, and vulnerability associated with a fiscal-financial planning model that considers the intervention of a line of credit to reduce the monthly VAT payment, within the legal framework.

Mathematical model

A probabilistic risk model is presented for companies with a fiscal-financial planning model that considers the intervention of a line of credit for the payment to suppliers to reduce the monthly VAT payment. The only requirements are time series, expected or estimated, of monthly sales and collections. Risk is defined as the product of the probability of a hazardous event and the company's vulnerability to that event.

Vulnerability is the expected difference in the company's monthly performance with and without hazardous conditions.

Cost function

Suppose that B_t and S_t are amounts that generate creditable VAT and input VAT collected in month t , respectively, where $t=1,2,\dots, n$. Thus, the VAT payable in the t -th month is:

$$D_t = \max\{[\theta/(1+\theta)](S_t - B_t), 0\}, \quad (1)$$

where θ is the VAT rate.

Now suppose, without loss of generality, that each month the line of credit is used to pay suppliers. Then, the function that determines the amount to be paid monthly is as follows:

$$C_t = \{D_t - [\theta/(1+\theta)]A_t\} + \tau A_{t-1}, \quad (2)$$

where D_t is given in (1), A_t is the amount used of the credit line in the t -th month, A_0 is a fixed and known value, and τ is the (fixed and known) interest rate of the credit line.

In this paper, A_t is defined as an amount whose applied VAT results in a proportion of the VAT payable; that is, $A_t = P_t[(1+\theta)/\theta]D_t$, where $0 \leq P_t \leq 1$. Thus, $P_t=0$ represents not using the credit line, and $P_t=1$ represents using the credit line for an amount that generates the total VAT payable in the t -th month. Therefore, when $t > 1$, C_t given in (2) can be written as:

$$C_t = D_t(1 - P_t) + \lambda D_{t-1}P_{t-1}, \quad (3)$$

where $\lambda = \tau[(1+\theta)/\theta]$. For $t=1$, $C_1 = D_1(1 - P_1) + \tau A_0$.

The mathematical expression that determines the total amount to be paid at the end of month $t=n$ is obtained by adding (3) over time; that is:

$$\sum_{t=1:n} C_t = \tau A_0 + \sum_{t=1:n} D_t(1 - P_t) + \lambda \sum_{t=1:n-1} D_t P_t, \quad (4)$$

For simplicity and practicality, it will be assumed that the credit line usage ratio, P_t , is unknown but fixed and constant during the study period (consisting of n months); that is, $P_t=P$ for all $t=1,\dots,n$. Therefore, the cost function given in (4) can be expressed as follows:

$$C_{1:n}(P) = \alpha - \beta P, \quad (5)$$

where $\alpha = \tau A_0 + \sum_{t=1:n} D_t$ y $\beta = D_n + (1 - \lambda) \sum_{t=1:n-1} D_t$. Note that (5) is a decreasing straight-line equation as a function of P. Thus, in the absence of risk, the minimum total amount payable at the end of the study period is reached at $P=1$.

Finally, from (5), the following expected total cost function is proposed in this paper:

$$C(P) = E[\alpha] - E[\beta]P, \tag{6}$$

where $E[\alpha] = \tau A_0 + \sum_{t=1:n} E[D_t]$ and $E[\beta] = E[D_n] + (1 - \lambda) \sum_{t=1:n-1} E[D_t]$. Note that estimating $E[\alpha]$ and $E[\beta]$, based on historical data, will make it possible to have an estimated version of (6). Later, the approach to estimate $C(P)$ will be presented in the subsection corresponding to the model implementation.

Vulnerability function

This paper considers $D_t=0$ as a hazardous event; that is, $(S_t - B_t) \leq 0$. If this event occurs in the t-th month, then the state of the indebtedness process, at time t+1, is composed of the use of the credit line at time t+1, which would be $A_{t+1} = P_{t+1}[(1+\theta)/\theta]D_{t+1}$ conditional on $D_t=0$, plus the debt of the previous period, which would be $(S_t - B_t)$. Note that no interest is paid since $D_t=0$. In terms of expected value, the indebtedness can be expressed as:

$$V^1_{t+1} = E[P_{t+1}[(1+\theta)/\theta]D_{t+1} | D_t=0] + E[S_t - B_t | D_t=0]. \tag{7}$$

On the other hand, if the dangerous event $D_t=0$ does not occur—that is, $D_t>0$ occurred—then the state of the borrowing process, at time t+1, is composed of the use of the credit line at time t+1, $A_{t+1} = P_{t+1}[(1+\theta)/\theta]D_{t+1}$ conditional on $D_t>0$, plus the interest corresponding to $A_t = P_t[(1+\theta)/\theta]D_t$. In terms of expected value, borrowing can now be expressed as:

$$V^0_{t+1} = E[P_{t+1}[(1+\theta)/\theta]D_{t+1} | D_t>0] + \tau E[P_t(S_t - B_t) | D_t>0]. \tag{8}$$

The vulnerability for month t+1, $V_{t+1}(P_{t+1})$, is defined as the difference between the expected indebtedness of the firm under non-hazardous and hazardous conditions; that is, $V_{t+1}(P_{t+1}) = V^0_{t+1} - V^1_{t+1}$, where V^0_{t+1} and V^1_{t+1} are given in (7) and (8), respectively. Thus, making $P_t=P$ for all t, the vulnerability function can be expressed as follows:

$$V_{t+1}(P) = P(1+\theta)/\theta \{ E[D_{t+1} | D_t>0] - E[D_{t+1} | D_t=0] \} + \tau PE[(S_t - B_t) | D_t>0] - E[S_t - B_t | D_t=0]. \tag{9}$$

Risk function

This paper considers a risk function that combines, in a multiplicative way, the exposure to the use of the credit line given by $E(P)=P$, the vulnerability $V_{t+1}(P)$, and the probability of the hazardous event $P[D_t=0]$; that is, the risk for month $t+1$ is $R_{t+1}(P)=E(P)P[D_t=0]V_{t+1}(P)$. Moreover, the total risk function is obtained by summing $R_{t+1}(P)$ over t , with $t=1, \dots, n-1$,

$$R(P) = \sum_{t=1:n-1} E(P)P[D_t=0]V_{t+1}(P). \tag{10}$$

Minimization of total impact

The total impact function of the fiscal-financial planning model is defined as the sum of the cost and risk functions. Thus, the total impact function is:

$$I(P) = C(P) + R(P), \tag{11}$$

where $C(P)$ and $R(P)$ are given in (6) and (10) respectively.

The mathematical problem of fiscal-financial planning consists of finding the value of P , in the interval $[0,1]$, that minimizes the total impact function given in (11); that is, finding the value P^* that satisfies the following:

$$P^* = \operatorname{argmin} I(P), \tag{12}$$

where $0 \leq P \leq 1$.

Implementation of the model

The following is a proposal of general steps to implement the fiscal-financial planning model. In addition, particular strategies to carry out the implementation are described.

Step 1: Generate predictions of B_t and S_t . It is proposed to apply parametric bootstrap using time series models from the family of integrated autoregressive and moving average (ARIMA) models, estimated based on historical data of monthly revenues and expenses of the company. In the free R

software, several libraries with functions allow these models to be fitted to observed data and simulate the estimated model. In particular, from the stats library, the function `arima()` can be used to fit the model, and `arima.sim()` to simulate M replications of B_t and S_t . The M replications (predictions) generated by the estimated model for the study period (for example, 6 months or 12 months), are denoted as B_{ij} and S_{ij} , where $t=1, \dots, n$ indicates the month, and $j=1, \dots, M$ is the index corresponding to the replication.

Step 2: Estimate the functions $C(P)$, $R(P)$, and $I(p)$. It is proposed to replace the theoretical quantities appearing in the model, such as $E[D_n]$, $E[D_{t+1} | D_t > 0]$, $E[D_{t+1} | D_t = 0]$, $E[(S_t - B_t) | D_t > 0]$, $E[S_t - B_t | D_t = 0]$ and $P[D_t = 0]$, by averages calculated based on B_{ij} and S_{ij} . Thus, the estimated versions of (6), (10), and (11), denoted by $\hat{C}(P)$, $\hat{R}(P)$, and $\hat{I}(P)$, are obtained. For example, the estimated cost function is:

$$\hat{C}(P) = \hat{E}[\alpha] - \hat{E}[\beta]P, \tag{13}$$

where $\hat{E}[\alpha] = \tau A_0 + \sum_{t=1:n} \hat{E}[D_t]$, $\hat{E}[\beta] = \hat{E}[D_n] + (1 - \lambda) \sum_{t=1:n-1} \hat{E}[D_t]$, $\hat{E}[D_t] = (1/M) \sum_{j=1:M} D_{tj}$ and $D_{tj} = \max\{[\theta/(1+\theta)](S_{tj} - B_{tj}), 0\}$. Note that, in the case of conditional expectation, the averages are calculated only with the cases that met the condition.

Step 3: Minimize the estimated total impact function. For the optimization process, it is proposed to use functions such as `optim()` or `optimize()` from the stats library of the free software R. It is also possible to use `which.min()` to identify the location of the minimum value of $\hat{I}(P)$ when P is defined as a vector object in R.

Data

Real historical data of monthly sales and collections recorded by a hardware wholesaler in Hermosillo, Sonora, Mexico, during 2018 were used. An equal amount was subtracted from the monthly collection amounts to generate a risky scenario to illustrate the potential of the methodology proposed in this article. It is worth mentioning that, for confidentiality reasons, the company's real name will not be mentioned.

On the other hand, a VAT rate of $\theta=0.16$ will be considered. For the interest rate associated with using the line of credit, a value of $\tau=0.0175$ will be taken. In addition, for simplicity, $A_0=0$ will be considered; that is, the process modeled here starts with no debt.

Simulation study

A simulation study was carried out to explore the behavior of the proposed model when faced with changes in the interest rate τ of the line of credit and the frequency of occurrence of hazardous events during the year. To control scenarios corresponding to the frequency of occurrence of hazardous events, the fiscal-financial planning model was implemented based on the observed data but subtracting an amount S_0 from the monthly collection amounts. In all cases, an autoregressive model of order 1, AR(1), was used to generate the predictions B_{ij} and S_{ij} , where $t=1, \dots, 12$ and $j=1, \dots, 5\ 000$. The values considered for S_0 range from 0 to 200 000 in increments of 50 000. On the other hand, the interest rate τ takes values ranging from 0.01 to 0.05 in increments of 0.01. Thus, in this simulation study, the value of P^* was calculated for the 25 possible combinations of (S_0, τ) .

Results

Case study

Figure 1 shows the trajectory corresponding to the difference between the observed values of S_t and B_t . In addition, this figure plots $M=5\ 000$ trajectories corresponding to the difference of the simulated predictions S_{ij} and B_{ij} , for a period of $n=12$ months. The predictions were based on an autoregressive model of order 1, AR(1). The model was fitted with both sales and collections data. The parameter estimates of the AR(1) model are presented in Table 1.

Table 1
AR(1) model parameter estimation, based on historical sales and collections data

Parameters of the model	S_t	B_t
Constant	7 780 449	6 052 806
Coefficient	0.7897332	0.7999303
Deviation	568 524	488 274.5

Source: created by the authors

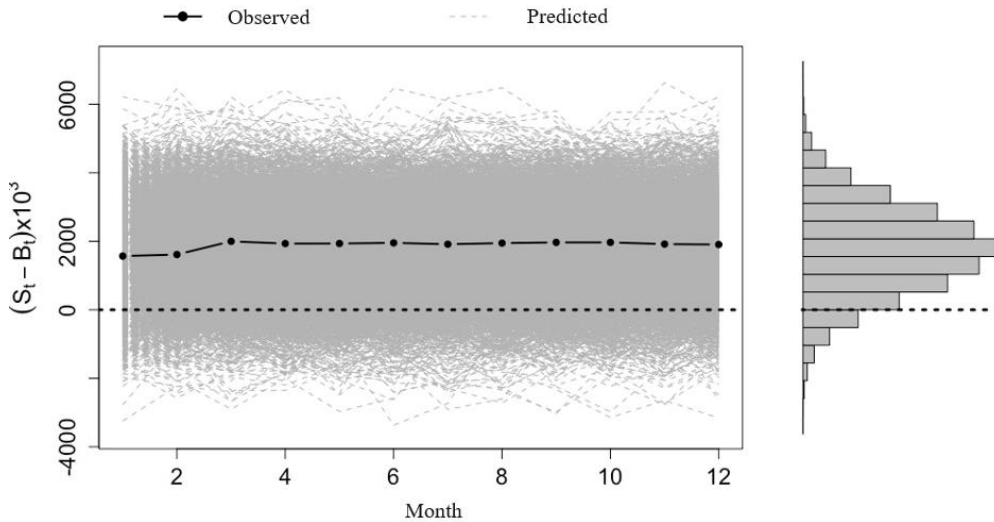


Figure 1. Trajectories corresponding to the difference between monthly sales and collections
 Source: created by the authors

Figure 1 shows the presence of trajectories that crossed the ordinates' Cartesian axis, represented by a dashed horizontal line. In addition, the histogram of the difference of the simulated predictions, shown in Figure 1, indicates that this is a case study where there is a non-depreciable probability of occurrence of hazardous events, $(S_t - B_t) \leq 0$ or equivalently $D_t=0$, throughout the year. In this case, the frequency of times the hazardous event occurred during the year was 8.38%. This result is interpreted as the risk that the flow generated by collections does not cover the debt contracted with the line of credit in the previous month.

Figure 2 presents, in the same graph, the estimated total cost function $\hat{C}(P)$, estimated total risk function $\check{R}(P)$, and estimated total impact function $\hat{I}(P)$. In addition, the location of the $P^*=0.82$ value is indicated, which indicates the optimal percentage of credit line usage in the fiscal-financial planning model applied here. Based on the data, these results suggest a monthly intervention of the line of credit for the payment to suppliers that covers 85% of the monthly VAT payment.

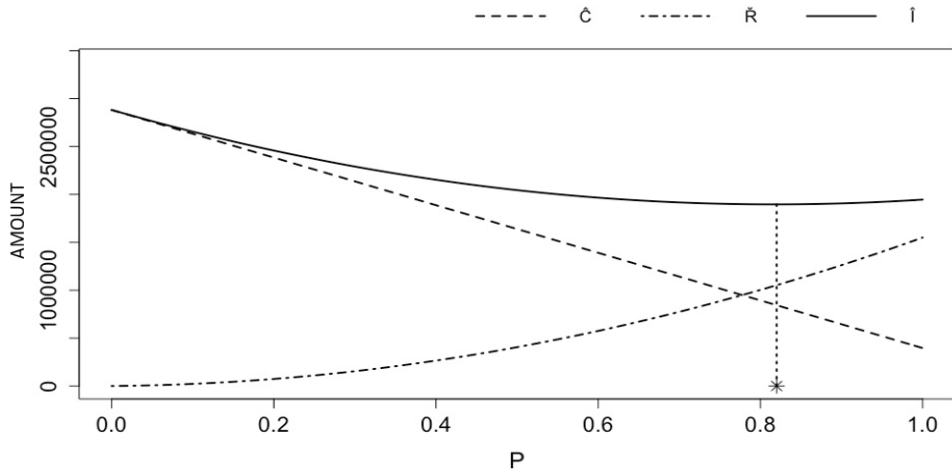


Figure 2. Estimation of the following functions: Total cost $\hat{C}(P)$; total risk $\hat{R}(P)$; total impact $\hat{I}(P)$. The location of the optimal percentage of use of the credit line is marked with an asterisk ($P^*=0.82$). Source: created by the authors

Simulation study

Table 2 presents the results obtained in the simulation study. This table shows that when the interest rate τ , associated with the credit line, is fixed, then P^* behaves in a monotonically decreasing way as a function of S_0 . Recall that S_0 is an amount subtracted from the observed monthly collections data to increase the frequency of times the hazard event, $(S_t - B_t) \leq 0$ or equivalently $D_t=0$, occurs during the year. Thus, the optimal percentage P^* of use of the credit line in the tax-financial planning model decreases as the frequency of occurrence of a hazard event—the amount that generates VAT collected is less than or equal to the amount that generates creditable VAT—increases.

Table 2

Value of P^* obtained for different combinations of (S_0, τ) , where τ is the interest rate of the credit line and S_0 is an amount subtracted from the observed monthly collections data. In all cases, an AR(1) model was used to generate the predictions B_{ij} and S_{ij} , where $t=1, \dots, 12$ and $j=1, \dots, 5\ 000$, which requires the calculation of P^* .

		S_0				
		0	50 000	100 000	150 000	200 000
τ	0.01	1	1	0.89	0.80	0.79
	0.02	0.97	0.91	0.83	0.75	0.68
	0.03	0.89	0.85	0.79	0.69	0.63
	0.04	0.78	0.72	0.70	0.66	0.56
	0.05	0.73	0.65	0.64	0.56	0.52

Source: created by the authors

Discussion

The results obtained in 2018 are compared with those of 2019 regarding the sales and collections gap and their impact on the risk sensitivity of the model proposed in this paper to determine the optimal percentage of credit line usage.

The difference between sales and collections is almost constant at around two million pesos throughout 2018, except for January and February, when it is below this amount. In contrast, Figure 3 illustrates that the difference between sales and collections is almost constant around one million nine hundred thousand pesos, during the first five months of 2019. Then, for the rest of the year, this difference increases to nearly two million eight hundred thousand pesos (an amount higher than what occurred in any month of the previous year; see Figure 1).

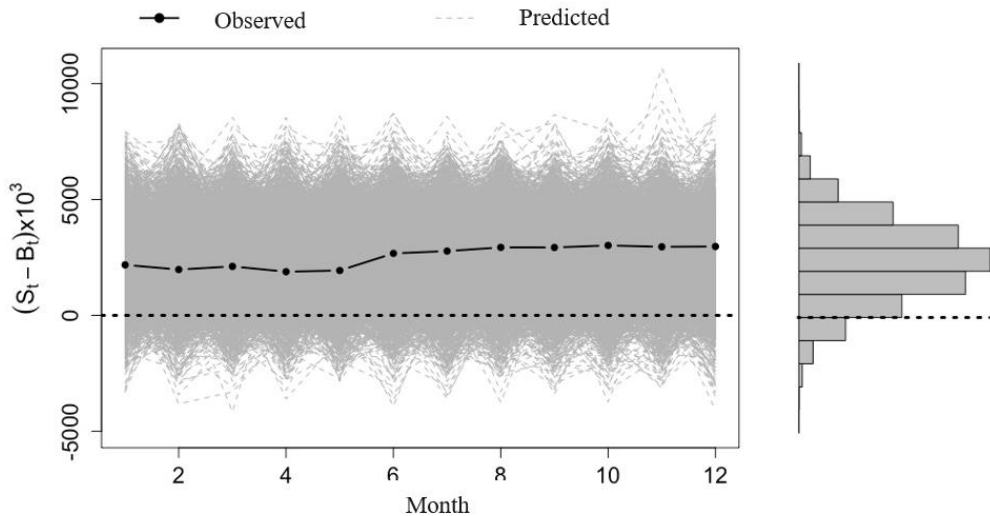


Figure 3. Trajectories corresponding to the difference between monthly sales and collections for 2019
 Source: created by the authors

From the second half of 2019, products began to be imported, financed by the line of credit. Likewise, the acquisition of transportation and warehouse equipment increased, which is why the gap between S_t and B_t compared to 2018 is greater since upon concluding the import process, paying for transportation equipment, and expanding the warehouse, a considerably higher creditable VAT was obtained than in 2018.

The frequency of times the hazardous event occurred during 2019 was 7.80%. Note that this is lower than that obtained for 2018, which was 8.38%. Thus, for 2019, the model indicates that the optimal

percentage use of the credit line is $P^*=1$; see Figure 4. This figure shows the trajectories of the estimated total cost, total risk, and total impact functions for 2019. It can be seen that the risk function, compared to that of 2018 presented in Figure 2, has a lower growth rate, leading to a decreasing impact function. Thus, the optimal percentage for 2019 yields a result of P^* equal to 1, which leads to contracting the credit line equivalent to one hundred percent of the tax gap, i.e., the difference between the VAT transferred and the creditable VAT. This contrast between the two periods shows the sensitivity of the results to the change in the tax gap and validates the consistency of the model in a real context.

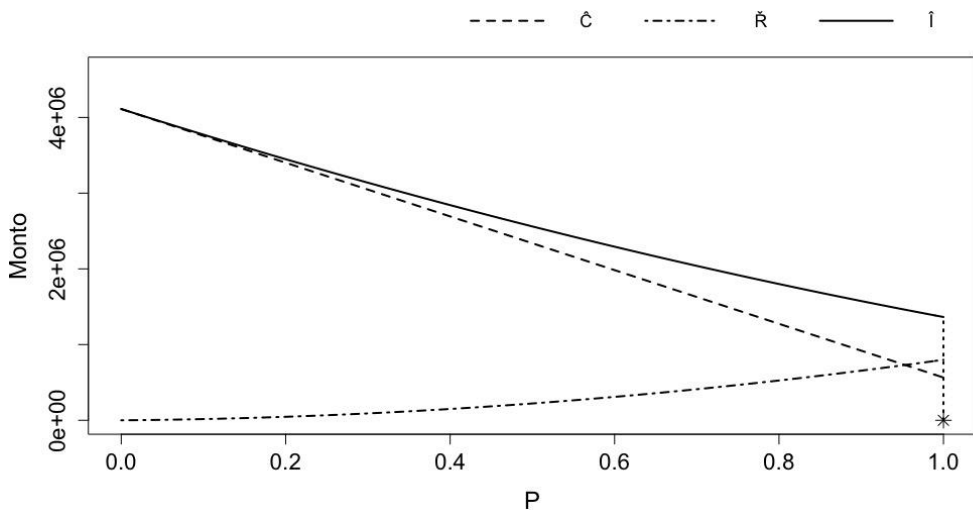


Figure 4. Estimation of the following functions: Total cost $\hat{C}(P)$; total risk $\check{R}(P)$; total impact $\hat{I}(P)$. The location of the optimal percentage of credit line usage ($P^*=1$) during 2019 is marked with an asterisk.

Source: created by the authors

Conclusions

This paper proposes a procedure to optimize the operation of a company and minimize a total impact function, including the cost, risk, and vulnerability associated with a tax planning model that considers the intervention of a line of credit for the efficient management of VAT payments. In particular, the proposed model is focused on reducing the impact on the financial flow of VAT due every month, considering four factors: VAT collections, VAT expenditures, line of credit as a source of financing, and the interest rate of the source of financing. The available credit line is used to make payments to suppliers, whether partial or total payments of invoices, which will generate a creditable VAT that helps to reduce the tax payable for the collection of the period in question.

The application of the model proposed in this paper was carried out with real data and simulation scenarios. In both cases, the model's behavior was satisfactory since it showed the sensitivity of its results to the risks implied by the fiscal gaps, generating optimal decisions ranging from using the entire credit line to more conservative situations in the presence of a narrower fiscal gap. Based on the above, it is concluded that the model is consistent and applicable in practice. It is worth mentioning that this model contributes to the scarce literature on fiscal decision making from a probabilistic and financial point of view since it generally focuses only on the accounting aspect.

The reduction of the tax burden is intended to provide improvements to the company, mainly to be able to allocate the savings to activities that help growth in the short and medium term, and by repeating the process during a certain period, it is possible to achieve financial strength and market positioning. In addition, this model can be applied to any company that is obliged to pay VAT according to the Law, regardless of its sector of economic activity. It is ideal for ongoing businesses searching for solidity and positioning since it is essential to have healthy liquid assets capable of meeting all types of expenses without neglecting compliance with tax obligations.

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