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Risk parity and diversification of investment portfolios

Paridad de riesgo y diversificación en portafolios de inversión

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Abstract

The Risk Parity (RP) model for optimal portfolios provides a valuable and robust alternative to the Markowitz's Mean-Variance (MV) model. In this paper, we propose a framework to apply RP approach for two different portfolios: i) for the U.S. stock market and ii) for an international portfolio with both developed and emerging markets, which includes Mexico and Brazil. To do that, we introduce an explanation of the fundamentals of the RP approach and PR-based investment strategies. In that sense, we apply the RP approach for the two portfolios, and we highlight the limitations of the MV model to construct diversified investment strategies given its problems of concentration and poor performance. Additionally, we extend the analysis with concentration metrics like the Herfindahl-Hirschman Index (HHI), allowing the consistency of the PR portfolios to be confirmed by reducing their rebalancing.

JEL Code: C61, D81, G11
Keywords: optimal portfolio; risk parity; diversification

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Resumen

El modelo de paridad de riesgo (PR) para la construcción de portafolios de inversión ofrece una alternativa valiosa y robusta frente al modelo media-varianza (MV) de Markowitz. En este trabajo se propone un marco de aplicación del enfoque de PR para dos portafolios diferentes: i) para el mercado de valores estadounidense y; ii) para un portafolio internacional con participación en los mercados desarrollados y emergentes en el que se incorporan México y Brasil. Para ello, introducimos una explicación de los fundamentos del enfoque de PR y de las estrategias de inversión basadas en PR. Luego, en la implementación del enfoque para los dos portafolios, se resaltan las limitaciones del modelo MV para el diseño de estrategias de inversión diversificadas, dado sus problemas de concentración y bajo desempeño. Además, se acompaña el ejercicio con métricas de concentración como el índice de Herfindahl-Hirschman (HHI) permitiendo confirmar la consistencia de los portafolios de PR al reducir su rebalanceo.

Código JEL: C61, D81, G11
Palabras clave: portafolio óptimo; paridad de riesgo; diversificación

Introduction

The mean-variance (MV) model formulated by Markowitz for the construction of diversified portfolios, although still valid today, presents some practical limitations that have led to the emergence of alternative approaches based on heuristic methods (Quian, 2005; Choueifaty & Coignard 2008; DeMiguel et al., 2009; Maillard et al., 2010; Roncalli & Weisang, 2016, among others). These approaches have helped overcome the problems of low diversification (or high concentration) and low performance presented by MV portfolios, given their high sensitivity and strong dependence on parameters estimated with historical information, as pointed out by Michaud (1989), Michaud (1998), Black and Litterman (1992), and Chopra and Ziemba (1993), among others.

The risk parity (RP) approach, introduced by Qian (2005), differs from the Markowitz MV model by considering each asset's contribution to the portfolio's total risk. To this end, this approach tries to make each asset contribute equally to the total portfolio risk (Roncalli, 2014; Roncalli & Weisang, 2016), i.e., the RP tries to make each asset or asset class contribute the same amount to the portfolio risk. In this approach, the RP is determined by the risk contribution (RC) measure, as it helps to understand how the risk characteristics of the portfolio change when the asset weights are adjusted. Therefore, the RP is part of the family of heuristic approaches for portfolio optimization[1], as stated by Maillard et al. (2010), Roncalli (2014), and Roncalli and Weisang (2016).

[1]Garlappi et al. (2007) and DeMiguel et al. (2009) provided several heuristic approaches for portfolio optimization that differ from RP.

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RP also represents a risk allocation approach, as Maillard et al. (2010) and Roncalli (2014) pointed out. According to the authors, RP allows risk to be decomposed into systematic and idiosyncratic components, leading to a risk budgeting process. As a result, RP helps to improve portfolio diversification and thus generates better performance while reducing the model's dependence on estimates of expected return parameters and covariances.

At the international level, there is extensive literature on the formal developments and applications of the RP approach, as well as the study of its advantages and benefits, compared to traditional models such as the Markowitz MV model (Fabozzi et al., 2021). After Qian (2005) introduced the RP approach, Maillard et al. (2010) presented general numerical solutions for constructing RP portfolios. Roncalli and Weisang (2016) implemented optimization exercises to build RP portfolios from the identification of risk factors or risk budgeting. Furthermore, Feng and Palomar (2015) formulated an algorithm that solves sequentially using a first-order convex approximation to the original nonconvex RP problem.

These developments were accompanied by previous work by Lohre et al. (2012), who analyzed uncorrelated sources of risk of the assets comprising the S&P 500 index through principal component analysis (PCA). In this work, the authors presented a method of constructing more diversified portfolios that align the principal components' contribution to the total investment risk.

Moreover, some extensions of the RP approach are found in the works of Bruder et al. (2022), Costa and Kwon (2020), and Bellini et al. (2021), among others. Bruder et al. (2022) recommended the incorporation of skewness in the RP portfolio instead of volatility, while Bellini et al. (2021) suggested an RP strategy based on expectiles as a measure of parity instead of volatility. Costa and Kwon (2020) presented a much more robust general reformulation of the RP approach by introducing ellipsoidal uncertainty sets and using estimation error minimization techniques for returns. Additionally, further extensions of the approach are identified in Chakravorty et al. (2019) and Lee and Sohn (2023) to design investment strategies. Chakravorty et al. (2019) designed active investment strategies from risk budgeting models, while Lee and Sohn (2023) extended the RP approach to combine investment strategies with style factors.

This paper reviews these advances in the RP approach and proposes a simple application model for portfolios with different characteristics. For this purpose, the RP model is implemented for: i) an investment portfolio with assets from the US stock market using the Dow Jones Industrial Average (DJI) as a reference; and ii) an international portfolio with participation of the representative indices of the US (DJI), UK (FTSE), Mexico (MXX), and Brazil (BVSP) markets. A comparison was made for both portfolios with the results of the traditional Markowitz MV model approach, taking different measures such as expected return, volatility, and Sharpe ratio. Moreover, concentration analysis of the portfolios using the Herfindahl-Hirschman index (HHI) is included. The paper contributes to the international literature on the RP approach to portfolio management in both developed and emerging markets.

This paper is organized into five sections, including this introduction. The second section presents the developments of the MV model for the construction of optimal portfolios. Afterward, the RP approach is introduced, considering its fundamentals and different formulations. In the fourth section, the implementation of the models is carried out and some comparisons of the results for the two proposed portfolios are made. Subsequently, some practical implications of the RP approach are presented. Finally, the conclusions and extensions of the work are shown.

Construction of optimal portfolios and Markowitz's MV model

Markowitz (1952, 1959) formulated the MV model by considering the expected value of asset returns ($\mu \in \mathbb{R}^{n\times 1}$) and the covariance matrix ($\Sigma \in \mathbb{R}^{n\times n}$) as inputs to that of the portfolio consisting of risky assets. If the expected return of the portfolio is determined by $\mu_p = w'\mu$, with variance equal to $\sigma_P^2 = w'\Sigma w$, with $w \in \mathbb{R}^{n\times 1}$ denoting the vector of weights (or percentage shares) of the assets, such that $w = (w_1, w_2, \ldots, w_n)'$, then the optimization problem is given by:

$$\min_{\{w\}} \{w'\Sigma\, w\} \text{ s.t. } w'1 = 1 \; w'\mu = \mu_0 \tag{1}$$

Where: $1 \in \mathbb{R}^{n\times 1}$ denotes a vector of ones and μ_0 represents the expected return of the portfolio with the lowest risk. The Markowitz optimization problem, assuming that asset returns follow a normal distribution, is solved by minimizing the portfolio risk measure (σ_P^2) for a given level of expected return (μ_0). This problem can also incorporate the constraint on negative weights, i.e., $w \geq 0$, if short sales are prohibited in the market. The MV model optimization problem is solved as a quadratic programming (QP) problem.

The Markowitz formulation laid the foundations for a deeper analysis of optimal portfolio construction that allows incorporating other restrictions to the optimization problem or even reformulating it for other risk measures such as semivariance (Sortino & Price, 1994), value at risk (VaR) or conditional value at risk (CVaR) (Uryasev & Rockafellar, 2001; Rockafellar & Uryasev, 2002). Nevertheless, this model has many problems, as pointed out by Michaud (1998), Best and Grauer (1991), and Black and Litterman (1992), among others. For example, using only historical data for the estimation of μ and Σ, it does not adequately incorporate the uncertainty of the estimated parameters, which generates very sensitive solutions. In addition, the MV model generates optimal portfolios that in practice present a high concentration in few assets, i.e., highly concentrated portfolios.

Risk Parity (RP) approach

Contribution to risk and basic formulations

The risk parity (RP) approach was introduced by Qian (2005)[2] and formally developed by Maillard et al. (2010) and Qian (2011). Unlike the Markowitz MV approach, which seeks to minimize the variance of the portfolio for an expected return level, the RP approach allows the construction of an investment portfolio in which all assets contribute the same amount (or proportion) to the total portfolio risk, i.e., all assets contribute equally to the portfolio risk.

RP emerged as an alternative to the MV model to construct "better" diversified portfolios, as stated by Fabozzi et al. (2021), because it considers the contribution to risk that each asset (CR_i) has in the investment portfolio. Since CR_i is determined by $w_i \frac{\partial \sigma}{\partial w_i}$, then, following Maillard et al. (2010) and Wu et al. (2020), the contribution of all assets to the portfolio risk, measured by their standard deviation σ_p, is obtained by[3]:

$$\sum_{i=1}^{n} w_i \frac{\partial \sigma_p}{\partial w_i} \tag{2}$$

Thus, for a set n of assets, a portfolio of equal risk contribution is the portfolio in which each asset has the same contribution to risk, i.e., risk parity should be satisfied:

$$w_i \frac{\partial \sigma_p}{\partial w_i} = w_j \frac{\partial \sigma_p}{\partial w_j} \; \forall \; i, j \tag{3}$$

[2]Although the term risk parity was coined by Qian (2005), the origin of risk parity dates back to the 1990s, when the Bridgewater mutual fund launched the first risk parity fund under the name All-Weather Fund. This strategy was based on a parity allocation with asset weights proportional to their inverse volatility. Subsequently, Qian (2005) and Maillard et al. (2010) developed a formal definition of risk parity that incorporates asset correlations based on the total risk contribution measure. For more details, see Rocanlli (2014) and Leon and Zapata (2023).

[3]Some extensions of this approach are found for downside risk measures, such as value-at-risk (VaR) or CVaR. For more details, see Boudt et al. (2013) and Haugh et al. (2017).

As Qian (2006) and Maillard et al. (2010) stated, there are different formulations to obtain the RP portfolio. The first formulation was developed by Qian (2005, 2006) and is known as naive RP (RPn), also called pseudo-RP, since it represents the simplest form of risk parity. The weights of the assets that comprise the RPn portfolio are obtained as a proportion of the inverse ratio of the risk of each asset measured by the standard deviations, as shown in Equation 3.

$$\mathrm{w} = \frac{\sigma^{-1}}{1'\sigma^{-1}} \tag{4}$$

In other words, the RPn consists in intuitively underweighting the riskier or more volatile assets and overweighting those of lower risk. Although this is a simple solution, it has the disadvantage of ignoring the covariances of the assets, which is critical for estimating the portfolio's total risk. As an alternative, vanilla RP (RPv)—also known as risk source parity—arises, as stated by Maillard et al. (2010) and Roncalli (2014). If the risk measure is the volatility of the portfolio, and analogously to what is indicated in Equation 3, one has $\mathrm{w_i}(\Sigma \mathrm{w})_\mathrm{i} = \mathrm{w_j}(\Sigma \mathrm{w})_\mathrm{j}$, then the RPv portfolio is obtained by solving the roots of the polynomial described in Equation 5.

$$w_i(\Sigma \mathrm{w})_i = b_i\ \mathrm{w}'\Sigma \mathrm{w} \tag{5}$$

In this formulation, a set of feasible constraints $\mathcal{W}$ are introduced, such as $1'\mathrm{w} = 1$ and $\mathrm{w} \geq 0$, such that $\mathrm{w} \in \mathcal{W}$, where:

$$\Sigma \mathrm{w} = \mathrm{b}/\mathrm{w} \tag{6}$$

As Maillard et al. (2010) suggested, the problem described in Equation 6 can be solved using numerical methods. They propose the following optimization problem:

$$\min_{\{\mathrm{w}\}} \sum\nolimits_{i=1}^{n} \sum\nolimits_{i=1}^{n} \left(w_i(\Sigma \mathrm{w})_i - w_j(\Sigma \mathrm{w})_j\right)^2 \text{ s.t. } \mathrm{w} \in \mathcal{W} \tag{7}$$

As the problem indicated in Equation 7 minimizes the squared differences between the CRs of all pairs of assets, then, for a feasible equal contribution portfolio, the optimal value of the function is zero. From this formulation, alternative solutions can be found in Roncalli (2014), Feng and Palomar (2015), Roncalli and Weisang (2016), and Wu et al. (2020), among others, who employ alternative

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methods. Moreover, as the RP only considers the portfolio risk measure, it can be seen as a risk management-oriented approach, as stated by Qian (2011) and Roncalli (2014), and allows the creation of better-diversified portfolios than the Markowitz MV model. That is, the RP can be seen as an allocation approach against the different sources of portfolio risk or risk budgeting.

In addition, the risk-return relation of the MV model can be adjusted to obtain an investment portfolio that achieves the highest degree of diversification. With this method, Choueifaty and Coignard (2008) developed the maximum diversification (MD) measure that depends solely on the portfolio's volatility. The MD measure is defined analogously to the Sharpe measure under the supposition that the portfolio return is directly proportional to its volatility. This redefinition of the risk-return relation makes it possible to extend this RP approach further.

Implementation of the model and analysis of results

In this section, the RP model is implemented for the construction of an investment portfolio: i) in the US stock market using the Dow Jones Industrial Average (DJIA) index, which is comprised of the 30 most important and representative industrial companies in the US stock market[4], and ii) for an international portfolio with the participation of the US (DJIA), Mexican (MXX), Brazilian (BVSP) and UK (FTSE) markets[5]. For both portfolios, a comparison is made with the results of the traditional Markowitz MV model approach, taking different measures such as expected return, volatility (or standard deviation), and Sharpe ratio. In addition, concentration analysis of the portfolios using the HHI index is included.

Data used

The RP model is implemented for the two proposed portfolios to show the advantages of the diversification approach for markets with different characteristics. The period of analysis was from January 2010 to December 2022, taking the adjusted closing prices of the stocks and of the representative indices of each market monthly.

[4]This market was chosen to reflect the main implications of the MV model. Since the DJIA is composed only of industrial companies, the correlations (or covariances) of these companies are high. Therefore, for a market with these characteristics, the MV model tends to present high concentration problems in the optimal composition of the portfolio.
[5]The implementation is done in R software. The codes and data used in this application are available upon request from the authors.

In the first case, the DJIA (the benchmark)[6] is used as the reference index for the US stock market. The 15 best-performing companies, as measured by the Sharpe ratio[7], are taken from this list of assets. Table 1 presents the expected return (average), volatility (standard deviation), and Sharpe ratio measures for the selected stocks and the index.

Table 1 shows the good performance of the selected companies during the analysis period. Additionally, Table 2 presents the same measures for the indices that comprise the international portfolio.

Table 1
Asset parameters - DJIA market

Assets	Expected return	Volatility	Sharpe Ratio
AAPL	0.0193	0.0785	0.2456
AMGN	0.0117	0.0635	0.1841
HD	0.0173	0.0607	0.2854
HON	0.0130	0.0585	0.2225
JNJ	0.0089	0.0434	0.2051
KO	0.0077	0.0489	0.1684
MCD	0.0116	0.0444	0.2608
MRK	0.0103	0.0504	0.1967
MSFT	0.0149	0.0621	0.2405
NKE	0.0136	0.0702	0.1930
PG	0.0084	0.0419	0.1998
TRV	0.0105	0.0548	0.1922
UNH	0.0195	0.0551	0.3546
V	0.0150	0.0582	0.2583
WMT	0.0082	0.0511	0.1597
DJIA	0.0047	0.0331	0.1417

Source: created by the authors

Table 2
Market parameters

Indices	Expected return	Volatility	Sharpe Ratio
DJIA	0.0047	0.0331	0.1417
MXX	0.0017	0.0323	0.0515
BVSP	0.0019	0.0504	0.0380
FTSE	0.0013	0.0285	0.0453

Source: created by the authors

[6]DOW is excluded from the list of DJIA companies as it does not present information for the entire analysis period. Table A1 shows the description of the selected companies.
[7]The purpose of this filter is to avoid companies with negative performance or high volatilities, which can lead to the construction of highly concentrated MV portfolios. By taking the full sample of DJIA companies, the optimal solution of the MV portfolio reaches a high number of excluded companies.

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Results for the US market portfolio; DJIA

The optimal MV, naive RP, and vanilla RP portfolios are constructed following the above formulations, taking only long positions (w ≥ 0). The results of the optimal weights of these portfolios are shown in Figure 1. The results confirm the significant differences in the composition of the portfolios of the MV model and the naive RP (RPn) and vanilla RP (RPv) models. There, the lower diversification of the MV portfolio is highlighted, in which not only high participation of a single asset (PGT) is identified but also the exclusion of five assets (AAPL, AMGN, HD, and V), although the reduced participation of assets such as NKE and KO is evident. On the other hand, the RPn and RPv portfolios present a much more homogeneous composition without excluding any asset. This shows the greater diversification found in the RP portfolios.

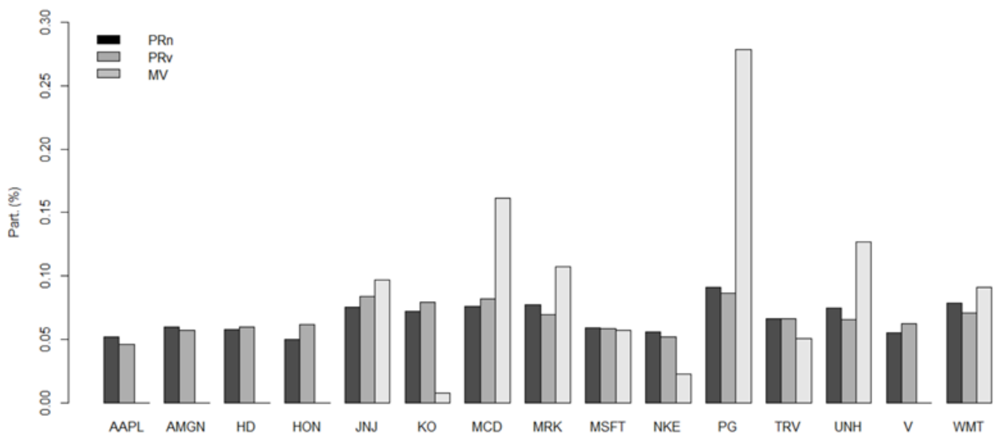

Figure 1. Optimum weights of the MV and RP portfolios
Source: created by the authors

On the other hand, in terms of performance, there is a notable improvement in the results of the RP portfolios (RPn and RPv) concerning the MV portfolio and the DJIA index, as shown in Table 3 and Figure 2. First, for the analysis period, higher expected returns and a better risk-adjusted return, measured by the Sharpe ratio, are found for the two RP portfolios.

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Table 3
Results of optimal portfolios and the DJIA index

	PRn	PRv	MV	DJIA
Return:	0.0122	0.0123	0.0111	0.0047
Volatility:	0.0347	0.0345	0.0318	0.0331
Sharpe ratio:	0.3518	0.3561	0.3508	0.1417
Num. Assets	15	15	10	29

Source: created by the authors

Figure 2, which presents the historical behavior of the accumulated returns for the optimal and DJIA portfolios, shows a better result for the RP portfolios, with no major difference for RPn and RPv. These results confirm the advantages of the RP models over the traditional MV model for constructing more diversified and better-performing portfolios. Additionally, to carry out a much more robust exercise to verify the results and to show the consistency of the RP portfolios compared to the MV portfolio, a monthly rebalancing exercise of the portfolios is carried out, taking a 60-month moving window.

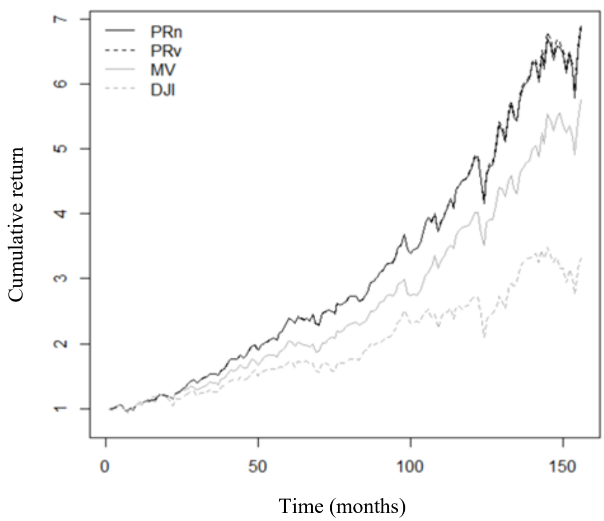

Figure 2. Cumulative return of the portfolios
Source: created by the authors

Considering the small differences between the RPn and RPv portfolios, only the RPn portfolio is used for this exercise. Based on these results, a new performance and concentration assessment is performed for all the portfolios. As a result, 85 portfolio updates are obtained for the entire analysis period. Figure 3 shows the rebalancing or updating processes in the composition of each portfolio (MV, RP, and RBP).

Figure 3 shows the frequent changes in the MV portfolio, which confirms the sensitivity problem of this model regarding the estimated parameters, as mentioned above. The exclusion of a large part of the portfolio's assets is also identified. In the case of the RPn portfolio, the changes from one period to another are minimal or even null in some cases, which confirms the greater consistency of this approach compared to the MV model.

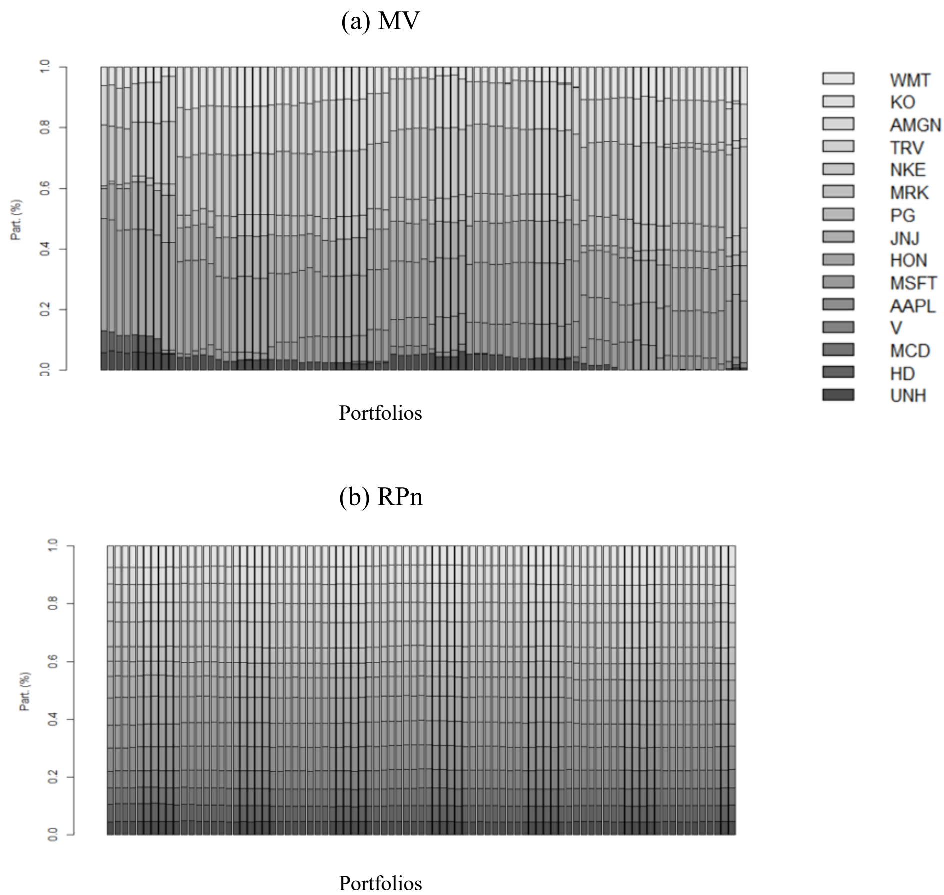

Figure 3. Rebalancing of portfolio weights
Source: created by the authors

Figure 4 shows the portfolios' expected return and Sharpe ratio calculations for each update over the analysis period. A notable advantage is observed for the expected return measure of the RPn portfolio and a higher Sharpe ratio in almost all periods.

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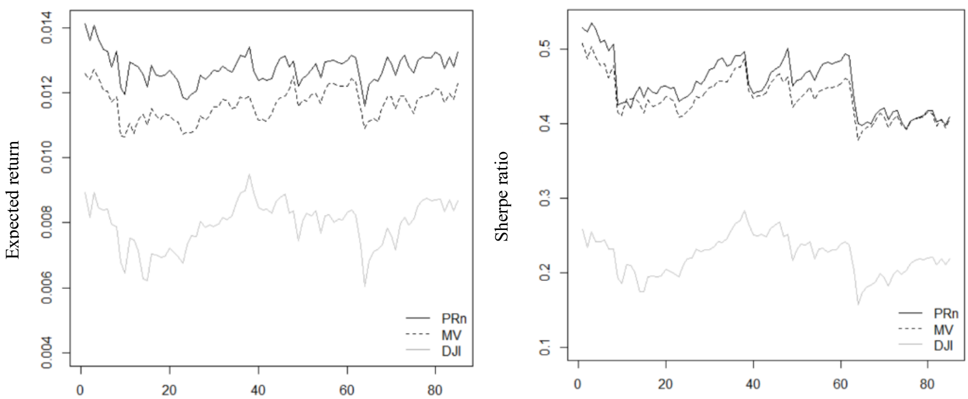
Finally, this consistency of the RP portfolios can also be confirmed by using a concentration indicator. For this, the HHI index[8] is used, as suggested by Leon and Zapata (2023) and Zapata et al. (2023), measured as the sum of the squares of weights of each n assets that make up the portfolio:

$$HHI = \sum_{i=1}^{n} {w_i}^2 \tag{8}$$



(a) Expected returns (b) Sharpe ratio

Figure 4. Portfolio performance during rebalancing

The HHI index is calculated for all the portfolios from the previous rebalancing exercise. Figure 4 shows the index calculated for the 85 moving portfolios. While the HHI indicator calculated for the RPn portfolio is around 690, this same indicator for the MV portfolio is between 1500-2300 and presents a high sensitivity, reflecting the radical composition changes when the data sample changes.

[8]An alternative formulation can also be implemented using the Gini coefficient, as suggested by Lohre et al. (2012).

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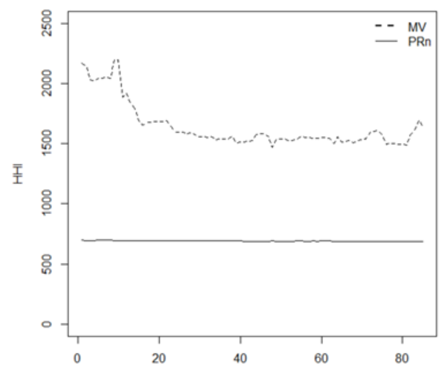
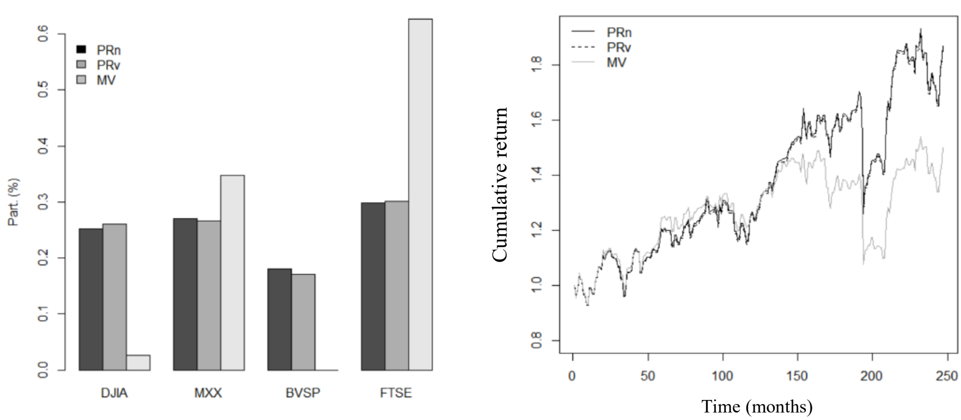


Figure 5. HHI index of MV and RPn portfolios
Source: created by the authors

Results for the international portfolio; DJI, MXX, BVSP, and FTSE

The optimal portfolios MV, naive RP, and vanilla RP are constructed following the same formulation as above, this time for an international portfolio that combines several markets. The results regarding the composition of the optimal portfolios (optimal weights) and their performance are shown in Figure 6 and Table 4.



(a) Optimal portfolio weights

(b) Cumulative return on portfolios

Figure 6. Portfolio results: composition and cumulative performance
Source: created by the authors

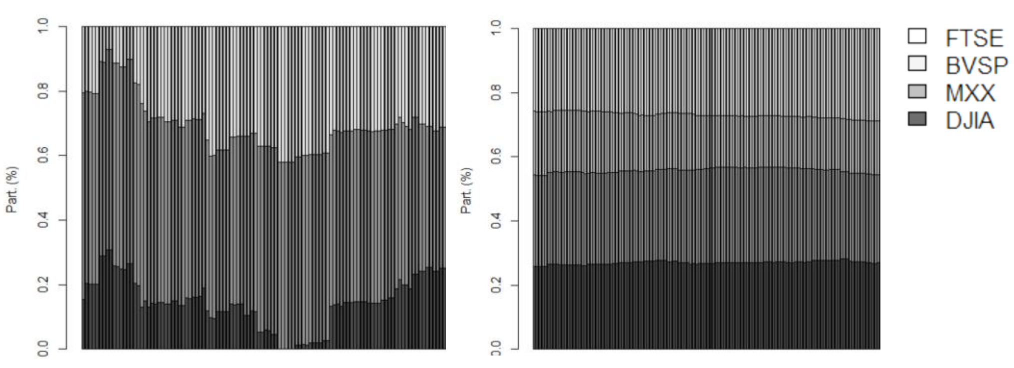
Table 4
Results of optimal portfolios

	PRn	PRv	MV
Return:	0.0024	0.0024	0.0015
Volatility:	0.0289	0.0290	0.0267
Sharpe ratio:	0.0823	0.0812	0.0565

Source: created by the authors

Once again, the results confirm the significant differences in the composition of the portfolios and the high concentration presented by the MV model compared to the alternative formulations RPn and RPv. A lower diversification of the MV portfolio stands out, with more than 60% participation in the FTSE market, the exclusion of BVSP, and a low participation of DJIA. Furthermore, regarding historical performance, the RP portfolios achieve a better result, as shown in Figure 6b and Table 4. The RPn and RPv portfolios achieve higher expected returns and Sharpe ratios despite the sharp drop in all of them, which corresponds to the period of the COVID-19 pandemic. These results confirm the advantage of the RP model for constructing more diversified and better-performing portfolios.

Furthermore, substantial changes are also found in the MV portfolio when performing the monthly rebalancing exercise of the portfolios, as shown in Figure 7a. In the 85 portfolio updates for the entire analysis period, the high sensitivity experienced by all markets is observed, while the rebalancing of the PRn portfolios is minimal.

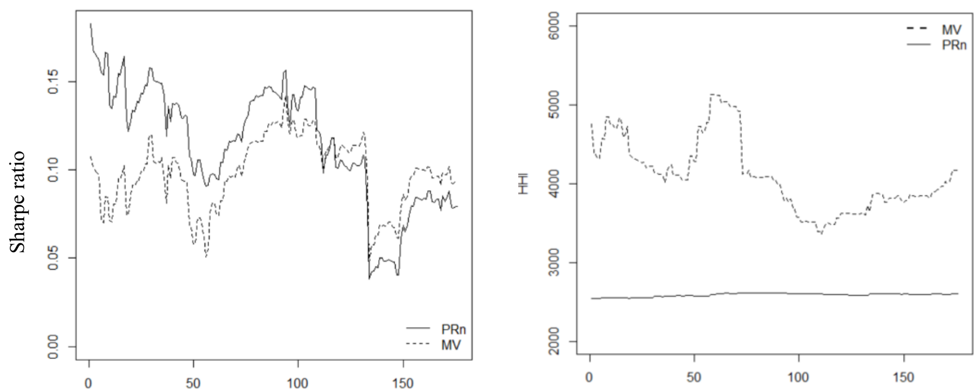


(a) MV (b) RPn

Figure 7. Rebalancing of portfolio weights
Source: created by the authors

Figure 3 shows the frequent changes in the MV portfolio, confirming the sensitivity problem of this model regarding the estimated parameters, as mentioned above. In contrast, for the RPn portfolio, changes from one period to the next are minimal or even null.

Finally, to confirm the greater consistency of this approach versus the MV model, Figure 8 shows the Sharpe ratio calculations of the portfolios for each update or rebalancing. A higher Sharpe ratio is observed in the RPn portfolio, except for some months at the end of the analysis period. Likewise, the HHI index for the 85 moving portfolios, as in the previous portfolio, shows a lower degree of concentration for the RPn portfolio than the MV portfolio. While for the RPn portfolio this indicator is below 2600 and shows very small changes, for the MV portfolio the indicator ranges between 3500 and 5000.



(a) Sharpe ratio (b) HHI index of the portfolios

Figure 8. Portfolio performance in rebalancing and HHI index

Discussion and practical implications

The risk-parity (RP) approach represents an advance on the Markowitz MV model as it not only allows the creation of diversified investment portfolios but also improves risk management efficiently. This result is achieved using intuitive, simple, and easy-to-implement optimization techniques. Moreover, by reducing the frequency of rebalancing, this approach allows investors to maintain a consistent strategy in the long run, which can help reduce transaction costs in the market. This can certainly be an important advantage for investment management professionals.

While these results are comparable with robust portfolio optimization approaches, using heuristic methods may generate some limitations that should not be overlooked, as Feng and Palomar (2015) and Costa and Kwon (2020) pointed out. When implementing heuristic methods, verifying that the solution is optimal (a global optimum) is impossible. Therefore, introducing uncertainty sets can help further reinforce the approach. Nonetheless, incorporating these techniques to strengthen the approach can lead to much greater mathematical complexity.

Conclusions

In this study, the RP approach was implemented to construct optimal investment portfolios to overcome the diverse limitations of the Markowitz MV model. This new RP approach allowed the construction of a much more robust portfolio for the two proposed applications, and the results overcome the sensitivity and diversification problems usually present in the MV model. Thus, the RP approach allows the creation of more consistent portfolios since it minimizes rebalancing over the analyzed period and achieves better diversification levels and performance. Therefore, the proposed RP approach offers advantages over the Markowitz MV model and overcomes its main limitations.

The advantage of the proposed model is that it can be easily replicated for different markets and asset classes. In future work, it is recommended to adopt alternative approaches to incorporate risk budgeting, as it provides a comprehensive view of risk attribution in the portfolio. It is also recommended to evaluate the sensitivity of the approach to other asset classes, such as commodities or fixed income securities. These extensions can contribute to the analysis derived from the RP approach with different asset classes and thus offer important developments for the financial industry.

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Annex

Table A1
List of selected DJIA index companies

Symbol	Company name	Sector
AAPL	Apple Inc.	Technology Services
AMGN	Amgen Inc.	Technology Services
HD	The Home Depot	Retail consumption
HON	Honeywell International Inc.	Technology Services
JNJ	Johnson & Johnson	Pharmaceutical industry
KO	The Coca-Cola Company	Beverages
MCD	McDonald’s	Fast-food restaurants
MRK	Merck Sharp & Dohme	Pharmaceutical industry
MSFT	Microsoft Corporation	Technology Services
NKE	Nike Inc.	Textile industry
PG	Procter & Gamble	Consumer goods
TRV	The Travelers Companies	Insurance
UNH	United Healthcare	Health
V	Visa Inc.	Financial Services
WMT	Walmart	Retail consumption