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Hierarchization of PEMEX exploration and production budget allocations in terms of technical efficiency

Jerarquización de las asignaciones presupuestales de PEMEX exploración y producción en términos de eficiencia técnica

Ricardo Aceves García[1*], Zaida Estefanía Alarcón Bernal[1], Mayra Elizondo Cortés[1], Germán López Bautista[2]

[1]Universidad Nacional Autónoma de México, México
[2]Petróleos Mexicanos, México

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Abstract

Throughout the history of Mexico, Petróleos Mexicanos (PEMEX) has been part of the conceptualization of its oil industry and a fundamental element for its development. Traditionally, Pemex Exploration and Production (PEP) develops the analysis of its budget allocation for oil fields, using economic indicators such as: net present value, present value of investment, investment efficiency, break-even point and accumulated cash flow, the which it examines and weighs jointly to generate information and make decisions. However, due to the economic importance and operating characteristics, PEMEX requires that its critical budget allocation activities to the investment portfolio be carried out using methodologies that increase its rigorous analysis capacity by considering additional aspects to the economic indicators which could hide overinvestments, high operating expenses, and other bad practices. This work presents an alternative to generate information through a mathematical optimization analysis, which will allow PEP to develop a better evaluation of its investment portfolio for budget allocation, through the evaluation of the technical efficiency of its oilfields.

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*Corresponding author.
E-mail address: aceves@unam.mx (R. Aceves García).
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Resumen

A través de la historia de México, Petróleos Mexicanos (PEMEX) ha sido parte de la conceptualización de su industria petrolera y un elemento fundamental para su desarrollo. Tradicionalmente, Pemex Exploración y Producción (PEP) desarrolla el análisis de su asignación presupuestaria para los yacimientos petroleros, utilizando indicadores económicos como: valor presente neto, valor presente de inversión, eficiencia de la inversión, punto de equilibrio y flujo de efectivo acumulado, los cuales examina y pondera de manera conjunta para generar información y tomar decisiones. Sin embargo, debido a la importancia económica y características de operación, PEMEX requiere que sus actividades críticas de asignación del presupuesto al portafolio de inversiones, se realice mediante metodologías que incrementen su capacidad rigurosa de análisis al considerar aspectos adicionales a los indicadores económicos los cuales podrían ocultar sobreinversiones, altos gasto de operación, y otras malas prácticas. En este trabajo se presenta una alternativa para generar información mediante un análisis de optimización matemática, que permitirá a PEP desarrollar una mejor evaluación de su portafolio de inversiones para la asignación presupuestaria, a través de la evolución de la eficiencia técnica de sus yacimientos.

Código JEL: H54, Q35, H83, C14, C44, C67
Palabras clave: PEMEX; yacimientos petroleros; administración pública; DEA; evaluación de eficiencia

Introduction

Petróleos Mexicanos (PEMEX) is Mexico’s national oil company. As a state-owned enterprise, it is wholly owned by the federal government. It has its own legal personality and assets, as well as technical, operational, and managerial autonomy. This company plays a key role in strengthening Mexico’s energy security and sovereignty.

The purpose of Petróleos Mexicanos is to create economic value and increase the nation’s income, with a sense of equity and social and environmental responsibility. In fact, it is the largest contributor to the federal government’s income (8.4% of GDP in 2022) according to the Center for Economic and Budgetary Research, A.C. (CEIP; Spanish: Centro de Investigación Económica y Presupuestaria, A. C.) (2023).

Pemex carries out activities throughout the entire hydrocarbon value chain, including the exploration and extraction of oil and liquid and gaseous hydrocarbons; the processing of natural gas; the refining of crude oil; and the synthesis of gas liquids and petrochemicals; as well as their collection, treatment, storage, transportation, and marketing.

Revenues are derived from gasoline sales and crude oil exports, which account for more than 66.1%. In 2022 alone, PEMEX generated revenues of MXN 1.102854 trillion (PEMEX, 2022), representing more than 8.4 percentage points of Mexico’s gross domestic product (CIEP; Spanish: Centro de Investigación Económica y Presupuestaria, A.C.; 2023). Its main lines of business are considered to

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be: exploration, production, and marketing of crude oil and gas; refining, processing, and marketing of petroleum products, natural gas, petrochemicals, and sulfur; treatment, primary logistics, and transportation services and storage of petroleum products and petrochemicals; production and marketing of fertilizers. This is achieved through four production subsidiaries: Pemex Exploration and Production (PEP; Spanish: Pemex Exploración y producción), Pemex Industrial Transformation (PTI; Spanish: Pemex Transformación Industrial), Pemex Logistics (PL; Spanish: Pemex Logística), and Pemex Fertilizers (PF; Spanish: Pemex Fertilizantes).

Pemex Exploration and Production is a company that aims to generate economic value for Mexico by exploiting 261 production fields or oil allocations, which manage 6 792 wells in operation and 300 offshore platforms, which together generate a production of 1 785 thousand barrels per day (Tbd) of crude oil and condensates, and 4 768 million cubic feet per day (Mcfd) of natural gas (PEMEX, 2022): allocations that were granted to it with the energy reform in 2014. Given the strategic importance of decisions regarding the Pemex Exploration and Production (PEP) allocation portfolio, these decisions must be technically sound, not merely economically sound, to meet their objective. Therefore, the technical efficiency of each oil allocation will be analyzed using the Data Envelopment Analysis (DEA) model.

Current status of the analysis of the allocation portfolio at PEMEX exploration and production

Constant changes in Mexico's oil industry have led the area responsible for managing the investment portfolio, PEP, to implement tools to support the documentation and analysis of oil allocations, as part of a monitoring and control process.

Traditionally, portfolio analysis in PEP is carried out using methods that apply classical portfolio theory (López Bautista, 2020), using an economic evaluation based on Pre-Tax Net Present Value (VPN AI; Spanish: Valor Presente Neto Antes de Impuestos), Post-Tax Net Present Value (VPN DI; Spanish: Valor Presente Neto Después de Impuestos), Present Value of Investment (VPI; Spanish: Valor Presente de la Inversión), investment efficiency (VPN/VPI; Spanish: Eficiencia de Inversión), break-even point (BE), and accumulated cash flow, which are used to classify and weight oil allocations in order to determine an order and carry out budget allocation.

This analysis is performed to generate the marginal capital return curve, considering that returns will produce a certain investment in the future; returns that will depend on expected sales and costs during the useful life of the allocation or project, that is, on the expected economic profitability, which is called the Probable Return on Investment. All of this is based on the concepts of Keynes (2014), who establishes

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that a marginal capital efficiency curve links the volume of investment to the marginal efficiency or profitability of the investment. Considering that all companies have their own marginal capital efficiency curves, if all these curves are added together, a market curve can be obtained, called the Investment Demand Curve or Marginal Capital Efficiency Curve.

While this analysis provides a ranking of oil allocations based on the results of financial indicators and the marginal capital return curve, it also generates, on the one hand, oil allocations that, due to their low production levels or the results of the indicator, will be discarded or limited for resource allocation; and, on the other hand, allocations that result in the top positions in the ranking but may conceal overinvestment, high operating costs, or poor policies, among other negative aspects (López Bautista, 2020).

This study used information from the 2018 portfolio for a group of 368 extraction allocations located in different geographical areas of Mexico and operated through the Subdirectorates of Production for Shallow Water Blocks (Spanish: Subdirecciones de Producción Bloques Aguas Someras) 01, 02, South, and North (SPBAS01, SPBAS02, SPBSur, SPBNorte).

The study was conducted at the unit's request for 30 producing fields, as an example of the total of 368 allocations. The elements of this selection share homogeneous characteristics, which permit their comparability. For confidentiality reasons, the selected oil allocations will be referred to as Decision Making Units (DMUs).

Analysis of PEMEX exploration and production to prioritize allocations

The analysis carried out by PEP to prioritize allocations simultaneously uses a set of financial indicators that provide information and guidance for budgetary decisions. In this analysis and prioritization, PEP primarily uses the following financial indicators: Net Present Value before and after taxes, Investment Efficiency, Break-even Point before taxes, and Accumulated Cash Flow.

PEP's strategy for ranking oil allocations is as follows. First, they calculate each of the financial indicators for the different allocations; second, with each indicator calculated, the allocations are ranked; third, for each allocation, the values obtained from each previous ranking are added together; and fourth, the sum of each allocation is weighted and the allocations are reclassified based on the weighted sum (from lowest to highest), thereby obtaining the ranking of the PEP investment portfolio for the annual budget allocation.

Table 1 below shows the DMUs' positions (oil allocation) relative to each indicator and the final ranking.

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Table 1
Hierarchization of DMUs according to PEP analysis

DMU	VPN AI	VPN DI	VPN DI/VPI	BE AI	BE DI	FE-ACUM	Sum	Weighted sum	Position
1-DMU	1	1	5	6	6	1	20	0.0072	1
2-DMU	2	3	7	4	7	2	25	0.0090	2
3-DMU	3	30	25	23	27	3	111	0.0398	21
4-DMU	4	4	8	7	9	5	37	0.0133	4
5-DMU	5	20	18	19	25	7	94	0.0337	15
6-DMU	6	6	2	8	15	9	46	0.0165	5
7-DMU	7	8	6	9	21	14	65	0.0233	6
8-DMU	8	14	17	14	23	13	89	0.0319	14
9-DMU	9	5	26	24	12	4	80	0.0287	10
10-DMU	10	2	3	2	2	6	25	0.0090	3
11-DMU	11	11	10	5	11	17	65	0.0233	6
12-DMU	12	7	19	22	8	8	76	0.0272	9
13-DMU	13	10	29	29	16	10	107	0.0384	20
14-DMU	14	19	13	10	13	15	84	0.0301	11
15-DMU	15	15	9	11	19	16	85	0.0305	12
16-DMU	16	16	11	12	20	19	94	0.0337	15
17-DMU	17	9	24	26	14	11	101	0.0362	18
18-DMU	18	24	12	15	4	21	94	0.0337	15
19-DMU	19	22	15	18	28	22	124	0.0444	24
20-DMU	20	17	28	25	10	20	120	0.0430	22
21-DMU	21	18	1	3	3	24	70	0.0251	8
22-DMU	22	12	30	30	30	12	136	0.0487	26
23-DMU	23	13	22	21	5	18	102	0.0366	19
24-DMU	24	28	16	27	29	25	149	0.0534	29
25-DMU	25	23	21	13	17	23	122	0.0437	23
26-DMU	26	27	23	20	26	26	148	0.0530	28
27-DMU	27	26	20	16	24	27	140	0.0502	27
28-DMU	28	29	27	28	22	28	162	0.0581	30
29-DMU	29	21	4	1	1	29	85	0.0305	12
30-DMU	30	25	14	17	18	30	134	0.0480	25

VPN AI = Net Present Value before taxes; VPN DI = Net Present Value after taxes; VPN DI/VPI = Investment efficiency; BE AI = Break-even point before taxes; BE DI = Break-even point after taxes; FE ACUM = Accumulated cash flow
Source: created based on PEMEX data (2018) reported in López Bautista (2020)

Pemex Exploration and Production, as a productive subsidiary of the Mexican state, whose value chain is exploration, production, and refining through allocations conferred by the state, faces the essential challenge of having an investment portfolio that guarantees the company's profitability. Therefore, analyzing and evaluating each oil allocation based on financial indicators, as is traditionally done, provides profitability information that has little to do with the technical efficiency with which the allocations or production fields analyzed operate.

Thus, as a specialized company that is critical to the country's economic development, PEP needs to implement methodologies that enable it to develop skills, use tools, and apply techniques that allow it to strengthen its analysis by measuring the technical efficiency of each of its projects and thus ensure better performance in its operations. Nevertheless, when it comes to measuring technical efficiency, the issue is not at all simple because each project has its own unique characteristics, making it difficult to compare them to determine efficiency.

Review of the literature

Regarding the evaluation of technical efficiency using Data Envelopment Analysis (DEA) in the oil industry, Bezerra et al. (2017) conducted a review covering the period from 1990 to 2015. They conclude that over 25 years, only 43 articles applied DEA to the oil industry, with the main areas of study being refinery efficiency, environmental efficiency, and efficiency in environmental management measures.

In recent years, DEA implementation has continued, and it is widely used to measure the relative efficiency of refineries and oil companies.

The work carried out by Alidrisi (2019) evaluates 10 petrochemical companies in the Kingdom of Saudi Arabia using the DEA models of Banker (1984) and Charnes et al. (1978) to calculate technical and super efficiencies, and classifies them according to their relative performance. This evaluation is based solely on the companies' financial data. Due to data access and technical limitations in this study, a framework based on the hybrid DEA-MDS (multidimensional scaling) approach was successfully established to evaluate the efficiency of the petrochemical industry. The efficiency graphs obtained with DEA were compared with the Euclidean distance scatterplot from MDS. It was found that the two-dimensional positioning of the companies was consistent in both graphs, thus validating the DEA results.

The purpose of the work carried out by Vikas (2019) was to determine the levels of technical efficiency, pure technical efficiency, and scale efficiency for 22 companies in the oil and gas sector in India that are listed on the National Stock Exchange and have data available for the period 2013-2017. The purpose was to provide benchmarks for inefficient companies to achieve their level of efficiency. The results revealed that 59% of all companies were both technically and scale efficient, while 73% were only technically efficient. Nevertheless, eight companies, representing approximately 54% of production, showed scale inefficiencies.

Ohene-Asare et al. (2017) evaluate the effects of multinational operations on oil companies' performance. This is achieved by comparing the productive and scale efficiencies of state-owned and private oil companies, as well as state-owned and private multinationals. The study was conducted using annual data from 50 companies from 2001 to 2010. While this study has provided valuable insights using data from the upstream oil industry, a more comprehensive picture would be presented if both upstream and downstream business segments were considered.

On the other hand, the work by Barros and Antunes (2014) analyzes changes in the productivity of Angola's oil blocks, using two alternative DEA models that include the Luenberger indicator and the Malmquist index.

The study conducted by Dalei and Joshi (2020) examined the technical efficiency of 12 oil refineries in India between 2011 and 2016 using DEA under an input-oriented VRS model. Based on the evaluation, a Tobit regression model was used to identify four significant explanatory variables that account for variations in technical efficiency. The result enabled the definition of improvement objectives for refineries considered inefficient, and subsequently generated specific recommendations, including increasing refinery operating time and enhancing maintenance policies, standard operating procedures, and employee behavior. It was also possible to make recommendations for the use of renewable resources in electricity generation, highlighting that analyzing the technical efficiency of oil refineries can be vital for a country's strategic development.

In their work, Wang et al. (2022) use the DEA models of Charnes, Cooper, and Rhodes (CCR) and Banker, Charnes, and Cooper (BCC) to evaluate the relative ecological and operational efficiency of ten petrochemical companies in China. They highlight the suitability of DEA as a systematic method for addressing unexpected results that reflect negative environmental impacts and for avoiding the subjectivity inherent in determining the weight of criteria, as in the analytical hierarchy process (AHP) and fuzzy evaluation methods. In DEA, weights are calculated from the internal model. The results were used to analyze differences in scale and technical efficiency. Estimates of operational and ecological efficiency, as well as the identification of influencing factors for the companies, were performed using regression analysis to verify and validate the model. Companies that did not achieve the effectiveness values were identified, and suggestions were provided to improve ecological efficiency targets and create benchmarks for the chemical industry in implementing sustainable development strategies.

Meanwhile, Sanchez-Robles et al. (2022) examine efficiency in the oil industry across 300 European companies from 2010 to 2019 using DEA. Based on this, they analyze the relation between efficiency and economic and financial variables, making important findings that demonstrate the value of DEA as a tool that enables consistent analysis beyond the technique itself. For example, they can gain important knowledge, such as the fact that companies with low efficiency are not sustainable in the long run, with implications for the careful design of strategic decisions. In addition, they observe that such a strategy will not be the same for all companies, recognizing that the use of DEA, while providing relative results, can help to better understand the efficiency of organizations differentiated by sector.

DEA has typically been used to measure the productive efficiency of producers in converting inputs into desirable outcomes (products, sales, customers served). A parallel treatment considers both desirable and undesirable outcomes. Studies such as those by Chen and Wang (2024) and Hatami-Marbini, Arabmaldar, and Asu (2022) present a useful and versatile approach to DEA, utilizing it to assess the impact of considering undesirable outcomes on productive efficiency. The latter study focuses particularly on the oil industry. It presents an empirical study on oil refineries in situations of data uncertainty, considering CO2 emissions as an undesirable outcome. The objective is to analyze environmental efficiency and productivity across 25 countries from 2000 to 2018.

In the work of Tavana, M et al. (2019), a dynamic multi-objective DEA model is proposed to determine the weights of inputs and outputs for the entire planning horizon; the corresponding efficiency scores are calculated across multiple planning periods to incorporate the dynamic nature of inputs and outputs into the analysis. A compromise solution is defined using fuzzy mathematical programming to address the multi-objective problem. The resulting procedure enhances the discriminatory power of DEA models in dynamic environments, reducing both calculation time and the complexity of their implementation. For this study, it should be noted that the proposed model incorporates both technical and environmental criteria that are generally omitted in the literature when evaluating oil refineries, a practice often due to limited access to the required data.

The study by Oliveira et al. (2023) uses DEA to present refinery improvement targets based on efficiency indices. It also uses the DEA window (DEA-WA) model, a variant of DEA, to analyze refinery efficiency across different periods. DEA-WA is integrated with the Malmquist index and cluster analysis to evaluate efficiency and identify the factors that explain differences across refineries over time. The numerical evaluations are made with data collected from 12 Brazilian oil refineries between 2012 and 2020.

As the review of the work on the use of DEA in the oil industry shows, there are very few studies, and they are mainly focused on measuring refinery efficiency rather than allocating budgetary resources.

The objective of this research is to evaluate the relative technical efficiency of the different PEP oil allocations, using the DEA methodology to rank the allocations and comparing the results obtained by DEA with the financial analysis traditionally carried out at PEMEX. Based on the above, improvement policies are proposed to address inefficient allocations relative to the best policies of efficient units, aiming to generate a more robust investment portfolio for informed decision-making in budget allocation.

Analysis strategy

The DEA method is a linear programming technique initially developed by Charnes et al. (1978) to evaluate the efficiency of decision-making units (DMUs) based on observed input and output levels. It is often used to explore the structure of productive efficiency and identify factors that may influence it. The DEA technique determines a piecewise linear envelope of the surface, also known as the efficiency frontier. DMUs that lie on the efficient frontier are efficient and can be seen as "best policy" units relative to their peers; on the other hand, DMUs that are not on this frontier are inefficient, and the analysis also provides measures of their relative efficiency. At this point, it is important to note that each DMU is free to choose any combination of inputs and outputs to optimize its efficiency.

Several authors who publish on the subject conceptualize the DEA model as an activity analysis based on linear programming, citing Koopmans (1976); this makes sense because activity analysis is a mathematical modeling approach.

According to Bogetoft and Otto (2010), an activity analysis model begins by describing the various activities that an organization performs, i.e., the different machines or processes used to achieve a goal. In linear programming (LP), these processes are represented by column vectors or variables that define how inputs are transformed into outputs. Nevertheless, the question asked in activity analysis is: how intensively should the different activities be used?

What is to be determined with the intensity of use of activities is the set of technologies or production necessary to meet an objective, considering that the constraints in this case reflect the available resources and the distribution of different resources among activities. To this end, the issue of activity intensity becomes the weights to be determined for each activity or variable, referred to as technological coefficients.

Consequently, the basis of the DEA technique is production theory and the idea that every company has a common underlying technology T, which is rarely known. DEA solves this problem by estimating technology T* from historical or cross-sectional data observed on actual production activities.

Specifically, DEA is a technique aimed at analyzing efficient frontiers rather than the central trend. From this perspective, the strategy is particularly useful for discovering relations that would remain hidden to other methodologies, such as considering a variable that is determined by efficiency, i.e., a variable within one production unit that is more efficient than in another.

DEA technologies

The basic DEA models differ mainly in the assumptions made for technology T. The most important assumptions are:

- Free availability. It is possible to produce less with more; that is, $(x, y) \in T, x' \geq x; y' \leq y \Rightarrow (x', y') \in T$
- Convexity. Any weighted average of feasible production plans is also feasible: $(x, y) \in T, (x', y') \in T, \alpha \in [0,1] \Rightarrow \alpha(x, y) + (1 - \alpha)(x', y') \in T$

- $\gamma -$ return of scale. Production can be scaled with any of a set of given factors: $(x, y) \in T, k \in T(\gamma) \Rightarrow k * (x, y) \in T$
- Additivity, replicability. The sum of two feasible production plans is also feasible, that is: $(x, y) \in T, (x', y') \in T \Rightarrow (x + x', y + y') \in T$

The assumption of free availability stipulates that unnecessary inputs and unwanted outputs can be freely discarded. Except in some cases of joint production, for example, when contamination occurs alongside desired products, this is a safe and weak assumption. The term 'weak' indicates that this assumption is safe to fulfill most of the time, but also that it has less power in expanding the set of production possibilities. Strong assumptions establish the opposite.

The convexity assumption states that any weighted average (convex combination) of feasible production plans is also feasible. This consideration is analytically convenient, and economic models assume a certain degree of convexity. In small datasets, convexity has significant power.

The assumption of returns to scale suggests that some rescaling is possible. The weakest hypothesis is that there is no possibility of readjustment, and the strongest is that there are constant returns to scale. No rescaling is also referred to as variable returns to scale, to use common terminology. Meanwhile, it is possible to assume that any degree of reduction is possible, but not any degree of increase. This means that being small can be advantageous, but being large can be disadvantageous, i.e., it is possible to have decreasing returns to scale. The last and least used hypothesis, which is actually quite natural and attractive, is that of increasing (or non-increasing) returns to scale. The idea here is that being large can be an advantage, but that being small can be a disadvantage.

Finally, the additivity hypothesis states that, when feasible production plans exist, their sum is also feasible, which is a natural assumption. Unfortunately, additivity is a difficult assumption to work with and is therefore the least common of these.

Graphically, the four basic DEA models with a single input and output can be represented as shown in Figure 1, which shows the cases of CRS (constant returns to scale), VRS (variable returns to scale), IRS (increasing returns to scale), and DRS (decreasing returns to scale).

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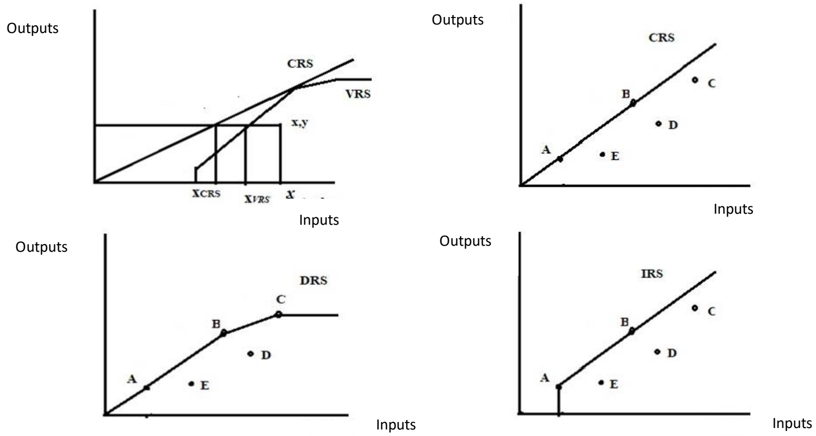

Figure 1. Technology of basic DEA models

Source: created by the authors based on Bogetoft and Otto (2010)

These relations suggest systematic differences in benchmarking results depending on the assumptions made a priori. Ideally, the choice of assumptions should be carefully argued and tested, if possible.

Consequently, the DEA approach involves searching for the smallest set that encompasses the input and output observations for all production units (the DMUs mentioned above). This also explains the name Data Envelopment Analysis.

Reference units and peer units

The DEA methodology identifies a specific reference unit, typically the weighted average of the existing units, which varies according to the unit being evaluated. Units with positive k weights are usually referred to as peer units, i.e.:

$$\text{Peer units} = k \in \{1, \ldots, K\} \,/\, \lambda^k > 0 \tag{1}$$

And, therefore, it is possible to establish that DEA explicitly identifies real peer units for each unit evaluated (Bogetoft & Otto, 2010).

Graphically, a reference unit is the unit located on the technological frontier onto which the evaluated unit is projected, and peer units are the units on the frontier that cover part of the frontier on which the reference unit is located. The reference unit and associated peer units are usually interpreted as indicating how the analyzed unit can be improved.

Although classical DEA models typically produce composite benchmark units, i.e., they use weighted means of data from several units, it remains true that DEA models use benchmarks based on a much smaller set of units compared to parametric models. Therefore, it can be argued that a distinctive advantage of DEA is that it provides explicit and comparable units.

In DEA models, the number of possible peers for analysis is equal to the number of inputs plus the number of outputs, except in the case of the CRS model, where there can generally be one less peer. According to linear programming theory, if there is an optimal solution, there is an optimal basis for which the number of positive variables is at most equal to the number of linear constraints. Therefore, only inputs and outputs that are definitely relevant should be included. If too many inputs and outputs are included, many units tend to become efficient, and the discriminatory power of the method is lost, i.e., it loses the ability to distinguish high performance from the rest. Thus, empirical rules have been suggested, one of which concerns the relation between the number of units to be analyzed and the number of inputs and outputs considered. According to Bogetoft and Otto (2010), this rule establishes that it is advisable for

$$k > 3(m + n) \; y \; k > (m * n) \tag{2}$$

Where:

$k =$ number of units to be analyzed;

$m =$ number of supplies; and

$n =$ number of products

These criteria are the basic ones, but other rules may be proposed.

DEA efficiency scale

Farrell's proposal (1957) views efficiency from a real rather than an ideal perspective, in which each DMU is evaluated relative to others within a representative, comparable group. In this way, efficiency measures will be relative rather than absolute, with the value achieved by a given production unit expressed as the deviation observed relative to those considered more efficient, given the available information. Therefore, a set of comparable DMUs must be examined, with the particularity that they use the same type of resources or factors to produce a set of similar or equivalent goods.

With the CRS model, and to a certain extent with the DRS and IRS models, the return to scale of properties is fixed, which is not the case for the VRS model. Therefore, it may be interesting to know what happens if the scale of a unit under analysis is changed. One possibility is that inputs and outputs will be scaled up or down by the same factor.

In a VRS model with a single input and output, it is easy to verify that as one moves along the input frontier, from the smallest to the largest, returns to scale first increase, then become constant, and finally decrease. Economically, this means that average output, i.e., the number of outputs per unit of input, first increases, then remains constant, and finally decreases (Figure 2).

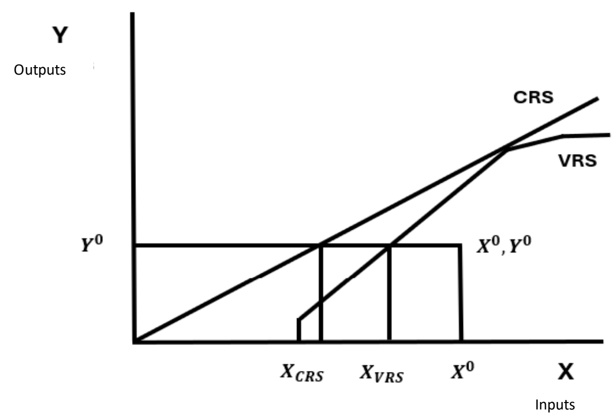

Figure 2. VRS technology efficiency scale
Source: created by the authors based on Bogetoft and Otto (2010)

In DEA, the concept of scale efficiency (SE) measures the loss incurred from not operating at an optimal scale. This indicator is calculated as the ratio between the input efficiency of a CRS model and that of a VRS model, i.e.:

$$SE(x^0, y^0) = \frac{(x^0, y^0; CRS)}{(x^0, y^0; VRS)} \tag{3}$$

Where:

x^0: input data in analysis;

y^0: output data in analysis

This indicator is never greater than 1 and equals 1 when VRS and CRS technologies coincide, that is, when a unit under analysis is operating at an optimal scale. The lower the SE value, the greater the loss from not achieving the higher average product that would be obtained at a more productive scale.

To better understand SE, it can be rewritten as follows:

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$$(x^0, y^0 CRS) = (x^0, y^0; VRS) * SE(x^0, y^0) \tag{4}$$

To this end, the strategy proposed by Farrell (1957) is a technique based on the concept of benchmarking, which means that efficiency (related to CRS technology) can be broken down into two components:

(i) pure (technical) efficiency, which measures the ability to use the best policies with VRS technology;

(ii) efficiency at scale, which measures the ability to operate where the average output per input group is maximized

Thus, it is possible to establish that the magnitude of the SE value can be calculated by comparing the indispensable inputs on the efficient VRS frontier and the indispensable inputs on the CRS frontier:

$$E(x^0, y^0; CRS) = \frac{\|x^{CRS}\|}{\|x^0\|} = \frac{\|x^{CRS}\|}{\|x^{VRS}\|} * \frac{\|x^{VRS}\|}{\|x^0\|} = SE(x^0, y^0) * E(x^0, y^0; VRS) \tag{5}$$

The efficiency scale SE measures how close the DMU is to the optimal scale size; the larger SE is, the closer the DMU is to the optimal scale. This information is interesting because it indicates the likely benefits of adjusting the DMU's scale. Nevertheless, it does not show to what extent an SE below 1 is due to the company being too small or too large. If the sum is less than 1, the DMU is below the optimal scale size, and if it is greater than 1, the DMU is above the optimal scale size.

At this point, it can be seen that conducting analyses based on an efficiency scale is attractive because it provides a measure of how much the DMU could benefit from adjusting its size, which is useful during the strategic planning process when deciding whether to adopt an expansion or contraction strategy.

Nonetheless, there are some caveats. First, the idea of adjusting scale may not be feasible because markets may not be competitive, and some companies may, for natural reasons, be unable to change their scale of operation. Second, the optimal scale depends on the exact direction in the input and output spaces. Therefore, it is not easy to derive simple guidelines in this regard.

DEA models for measuring efficiency

While efficiency gaps can be identified in individual performance measures, combining multiple measures in the final stage remains challenging. Therefore, benchmarking models are needed that can address multiple performance measures and provide an integrated benchmarking measure. This is because, once the frontier is established, it is possible to compare a set of new DMUs with the current frontier.

Nevertheless, when a new DMU exceeds the identified frontier, DEA generates a new frontier, so the same reference (frontier) is not available for other (new) DMUs.

Basic linear and fractional programming schemes for DEA models

To establish mathematical development, it will start with the formulation of fractional programming of the CCR model, in order to consider two relevant points:

1) Generalize the ratio of one output to another input;

2) Derive efficiency scores for each DMU relative to the performance of all DMUs.

Therefore, considering the model developed by Cooper (2011) for constant-scale input-oriented performance, which can be formulated as:

$$\text{Max } h_0(u,v) = \sum\nolimits_r {}^{u_r y_{r0}}\!/\!{}_{\sum_i v_i x_{i0}}$$

Subject to:

$$ {}^{\sum_r u_r y_{rj}}\!\Big/\!{}_{\sum_i v_i x_{ij}} \leq 1 \text{ para } j = 1, \dots, n$$

with

$$\frac{u_r}{\sum_{i=1}^{m} v_i x_{i0}}, \frac{v_i}{\sum_{i=1}^{m} v_i\, x_{i0}} \tag{6}$$

Where the variables y_{ri} and x_{ij} correspond to the output and input values, with $r = 1, \dots, s$ and $i = 1, \dots, m$ for each $j = 1, \dots, n$ of the different DMUs, and the variables y_{r0} and x_{i0} in the objective function, the outputs and inputs for DMU_0 being evaluated.

The problem represented by the fractional equation is convex but not linear. Applying the variable transformation proposed by Charnes and Cooper (1962) to convert the fractional problem into an equivalent linear programming problem called multiplicative, it is obtained that:

$$\text{Max } z = \sum\nolimits_{r=1}^{s} \mu_r y_{r0}$$

Subject to:

$$\sum\nolimits_{r=1}^{s} \mu_r y_{rj} - \sum\nolimits_{i=1}^{m} v_i x_{ij} \leq 0, \qquad j = 1, \dots, n$$

$$\sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\mu_r, v_i \geq \varepsilon > 0$$

(7)

Where:

y_{ri} *and* x_{ij} represent the quantities of outputs and inputs, with $r = 1, \dots, s$ and $i = 1, \dots, m$, for each $j = 1, \dots, n$ of the different DMUs;

μ_r and v_i are the weights or multipliers of the outputs and inputs, respectively;

y_{r0} and x_{i0} represent the output and input quantities of the evaluated unit;

$\varepsilon > 0$ guarantees that the solutions will be positive in all variables, so that each input and output must have some value, however small.

Furthermore, since it is known that each linear programming problem has another linear programming problem associated with it called a dual problem, it is possible to establish this dual problem, called an enveloping model, as:

$$Z_0^* = \min \theta - \varepsilon \left(\sum_{r=1}^{s} S_i^- + S_r^+ \right)$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \lambda_j + S_i^- = \theta x_{i0} \; i = 1, \dots, m$$

$$\sum_{j=1}^{n} y_r \lambda_j + S_r^+ = \; y_{r0} \; r = 1, \dots, s$$

$$\lambda_j, S_i^-, S_r^+ \geq 0, \forall \; i, j, r$$

(8)

Where:

λ_j are the dual variables of the problem;

S_i^- and S_r^+ are the slack variables of the inputs and outputs, $i = 1, \dots, m; \; r = 1, \dots, s <$;

θ is the efficiency of the unit evaluated;

$\varepsilon > 0$ is an element called non-Archimedean

Approaches (6) and (7) define efficiency by calculating the ratio of the objective of the DMU being evaluated. The incorporation of the Pareto-Koopmans efficiency definition (Alberto Jaime, 2016) is represented in model (8), which means that a DMU will be efficient if and only if $\theta^* = 1$ and all slack variables are zero; otherwise, the DMU is classified as inefficient.

An interesting aspect of this definition of efficiency is that if, at the optimum, $S_r^{+*} > 0$, it means that it will be possible to increase the output r of the DMU under evaluation by the amount given by the slack, whereby the DMU under evaluation should produce an output $(y_r + s_r^{+*})$ instead of the observed amount y_r. Similarly, if at the optimum $S_i^{-*} > 0$ for input i of the DMU under evaluation, it will be possible to reduce that input in the evaluated DMU by $(x_i - s_i^{-*})$ instead of using the quantity x_i.

DEA BCC or VRS model (with variable-scale efficiency)

The model used in this study is the BCC or VRS model proposed by Banker (1984), which considers variable returns to scale. This model provides a measure of how much the DMU could benefit by adjusting its size, which is valuable in strategic planning stages when deciding between an expansion or contraction strategy.

The DEA model, operating under variable returns to scale (VRS), offers numerous advantages when evaluating the efficiency of decision-making units (DMUs). It enables efficiency to be assessed in contexts where operating units vary significantly in size and capacity, providing a more accurate and equitable assessment. In addition, it facilitates the identification of economies and diseconomies of scale, which is valuable for strategic decision-making (expansion or contraction of operations). The VRS model also distinguishes between technical inefficiency (resulting from poor resource management) and scale inefficiency (resulting from operating at a suboptimal size), providing a more detailed and targeted analysis that helps decision-makers develop effective, targeted improvement strategies. For these reasons, the VRS model is considered the preferred benchmarking option in many practical scenarios. By not imposing the constant returns constraint, it can be applied to a wider range of contexts and sectors, making it more adaptable and useful for analyzing organizations with very diverse operational characteristics.

An important consideration when selecting a DEA model is that, in applications where the production factors (inputs) are not fully under the manager's control, output-oriented models are more appropriate. In contrast, if the results of the process (outputs) are determined by the managers' objectives rather than established based on observed best practice, it would be preferable to use input-oriented models (Sodani, 2011).

The BCC or VRS model is actually an extension of the CCR model. Therefore, its formulation is similar. The essential difference between these models is that BCC assumes variable returns to scale, whereas CCR assumes constant returns to scale. This difference is established by the convexity constraint associated with the BCC model (9).

$$Z_0^* = \min \theta - \varepsilon \left(\sum_{r=1}^{s} S_i^- + S_r^+ \right)$$

Subject to:

$$\sum_{j=1}^{n} x_{ij}\lambda_j + S_i^- = \theta x_{i0} \; i = 1, \dots, m$$

$$\sum_{j=1}^{n} y_r\lambda_j + S_r^+ = \; y_{r0} \; r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j \quad = 1 \; j = 1, \dots, n$$

$$\lambda_j, S_i^-, S_r^+ \geq 0, \forall \; i, j, r$$

(9)

Where $\sum_{j=1}^{n} \lambda_j \quad = 1; \; j = 1, \dots, n$ is the convexity constraint.

The DMU evaluated will be rated as efficient, according to Pareto-Koopmans efficiency, if and only if in the optimal solution $\theta = 1$ and the slack variables $S^{-*} = 0$ and $S^{+*} = 0$

The nature of the locally prevailing returns to scale for a DMU can be determined from model (9). Thus, if a sample of n DMUs is considered and it is assumed that DMU_0 meets the Pareto-Koopmans efficiency conditions, solving the problem given by model (9) for DMU_0 will produce the optimal values λ^*, such that:

- $\sum \lambda^* > 1$: Decreasing returns to scale prevail locally for DMU_0;
- $\sum \lambda^* = 1$: Constant returns to scale prevail locally for DMU_0;
- $\sum \lambda^* < 1$: Increasing returns to scale prevail locally for DMU_0.

Problem resolution

To carry out this work, information corresponding to the 2018 PEP investment portfolio was used for the group of 368 extraction allocations, which are located in different geographical areas of Mexico and are operated through the Subdirectorates of Production Shallow Water Blocks 01, 02, South, and North (SPBAS01, SPBAS02, SPBSur, SPBNorte) (López Bautista, 2020). For this analysis, the agency proposed selecting 30 allocations based on Net Present Value before taxes (VPN AI), as this indicator is considered the most important in this group of allocations. The initial ranking for the 30 selected DMUs is shown in Table 2.

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Table 2
Ranking of DMUs based on their VPN AI (MMXN)

Position via VPN	DMU	VPN AI	Position via VPN	DMU	VPN AI
1	1-DMU	441 882	16	16-DMU	47 429
2	2-DMU	251 937	17	17-DMU	45 380
3	3-DMU	232 699	18	18-DMU	40 838
4	4-DMU	215 256	19	19-DMU	38 880
5	5-DMU	132 616	20	20-DMU	31 730
6	6-DMU	107 990	21	21-DMU	30 726
7	7-DMU	107 049	22	22-DMU	29 264
8	8-DMU	99 994	23	23-DMU	29 040
9	9-DMU	89 925	24	24-DMU	27 238
10	10-DMU	85 308	25	25-DMU	26 513
11	11-DMU	72 545	26	26-DMU	21 871
12	12-DMU	58 630	27	27-DMU	18 872
13	13-DMU	54 788	28	28-DMU	14 008
14	14-DMU	51 401	29	29-DMU	6 897
15	15-DMU	49 574	30	30-DMU	6 552

Source: created based on PEMEX data (2018) reported in López Bautista (2020)

The proposal was to use the DEA model with BCC variable-scale efficiency, conducting the analysis based on inputs, since the production factors (inputs) are completely under the administrators' control. The input variables for each DMU are the investment or budget allocated and the annual operating spending in billions of pesos for each allocation, and the output variables are the annual oil production and annual gas production, in millions of barrels of crude oil per day (MBD) and billions of cubic feet of gas per day (MMPCD), respectively.

The data used for each DMU in the DEA relative efficiency analysis are presented in Table 3. This table also indicates that DMU22 does not produce oil, so it is assigned a small value for those data.

Table 3
Data used as inputs and outputs for analysis

DMU	Inputs		Outputs	
	Investment	Operating Expenses	Oil Production	Gas Production
1DMU	155785	81489	1220826	463273
2DMU	124 696	862 398	102 043	119 753
3DMU	416 626	348 326	1 700 383	3 522 258
4DMU	88 085	43 929	602 843	279 350
5DMU	111 135	69 268	461 578	112 838
6DMU	26 242	15 216	189 970	243 843
7DMU	41 334	17 989	236 899	157 790
8DMU	92 350	22 831	289 842	246 491
9DMU	146 523	33 479	420 975	322 511
10DMU	26 496	24 223	115 354	1 410 747
11DMU	32 073	14 827	126 579	411 081
12DMU	55 217	50 579	240 657	371 240
13DMU	124 927	23 280	252 374	426 033
14DMU	32 147	162 077	198 701	22 731
15DMU	17 776	10 172	121 881	78 567
16DMU	15 141	4 801	77 974	141 419
17DMU	57 006	37 445	173 227	472 601

DMU	Inputs		Outputs	
	Investment	Operating Expenses	Oil Production	Gas Production
1DMU	155785	81489	1220826	463273
2DMU	124 696	862 398	102 043	119 753
3DMU	416 626	348 326	1 700 383	3 522 258
18DMU	17 365	70 088	83 184	908 152
19DMU	30 029	12 117	115 097	64 082
20DMU	68 425	13 545	118 821	387 982
21DMU	5 236	9 206	33 324	270 462
22DMU	123 245	28 741	100	1 140 092
23DMU	27 701	26 385	116 183	281 574
24DMU	22 991	47 516	102 059	79 379
25DMU	29 319	123 203	140 295	142 26
26DMU	46 240	3 818	78 611	359 27
27DMU	25 477	4 351	92 088	272 79
28DMU	24 189	3 315	48 934	714 08
29DMU	38 567	27 148	108 207	3 365 31
30DMU	15 868	22 508	68 931	721 28

Source: created based on data from PEMEX (2018)

The BCC model was implemented for inputs in an application with R-Shiny, to provide an easily accessible tool at no major cost to the company

Table 4
Efficiency analysis results

DMU	Efficiency	Investment Slack	Slack Expenditure Operation	Lamdasum	Returns
1DMU	1	0	0	1	Constant
2DMU	1	0	4.18672E-07	1	Constant
10DMU	1	0	0	1	Constant
16DMU	1	0	0	1	Constant
21DMU	1	0	0	1	Constant
26DMU	1	0	0	1	Constant
27DMU	1	0	0	1	Constant
18DMU	0.989417	0	50119.57182	1.28739	Decreasing
6DMU	0.978516	0	0	0.29684	Increasing
4DMU	0.909399	0	0	0.97552	Increasing
15DMU	0.882501	0	0	0.21473	Increasing
28DMU	0.858775	6234.201896	0	0.62624	Increasing
7DMU	0.852157	0	0	0.89169	Increasing
14DMU	0.76129	0	0	0.18478	Increasing
11DMU	0.740564	0	0	1.25613	Decreasing
20DMU	0.721542	25710.72711	0	1.45503	Decreasing
8DMU	0.70319	0	0	2.89793	Decreasing
22DMU	0.681112	62531.16314	0	0.80815	Increasing
9DMU	0.67579	0	0	4.35744	Decreasing
13DMU	0.654419	23544.96276	0	3.2342	Decreasing
19DMU	0.601878	0	0	0.4364	Increasing
25DMU	0.593201	0	0	0.12766	Increasing
12DMU	0.574854	0	0	1.26119	Decreasing
23DMU	0.567729	0	0	0.99233	Increasing
24DMU	0.567682	0	0	0.26498	Increasing
30DMU	0.562516	0	0	0.23993	Increasing
3DMU	0.547658	0	0	12.11254	Decreasing
5DMU	0.529565	0	0	0.37926	Increasing
17DMU	0.447665	0	0	0.46989	Increasing

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DMU	Efficiency	Investment Slack	Slack Expenditure Operation	Lamdasum	Returns
1DMU	1	0	0	1	Constant
2DMU	1	0	4.18672E-07	1	Constant
10DMU	1	0	0	1	Constant
16DMU	1	0	0	1	Constant
29DMU	0.419167	0	0	0.40583	Increasing

Source: created by the authors

Based on the DEA results in Table 4, the following 7 DMUs are identified as efficient, as shown in Table 5.

Table 5
Efficient DMUs

DMU	Efficiency
1DMU	1.00
2DMU	1.00
10DMU	1.00
16DMU	1.00
21DMU	1.00
26DMU	1.00
27DMU	1.00

Source: created by the authors

Nevertheless, according to the Pareto-Koopmans efficiency concept, of these seven DMUs considered on the efficiency frontier, DMU2 should be considered inefficient because it has a slack different from zero (4.18672E-07) for input 2, although this value is very small (Operating Expenses).

Considering the other 23 DMUs as inefficient, they are reflected in Table 6.

Table 6
Inefficient DMUs

DMU	Efficiency
18DMU	0.989417
6DMU	0.978516
4DMU	0.909399
15DMU	0.882501
28DMU	0.858775
7DMU	0.852157
14DMU	0.76129
11DMU	0.740564
20DMU	0.721542
8DMU	0.70319
22DMU	0.681112
9DMU	0.67579
13DMU	0.654419
19DMU	0.601878
25DMU	0.593201
12DMU	0.574854
23DMU	0.567729
24DMU	0.567682
30DMU	0.562516
3DMU	0.547658

Source: created by the authors

The graphical representation of the efficiency results obtained for the 30 oil wells analyzed is shown in Figure 3.

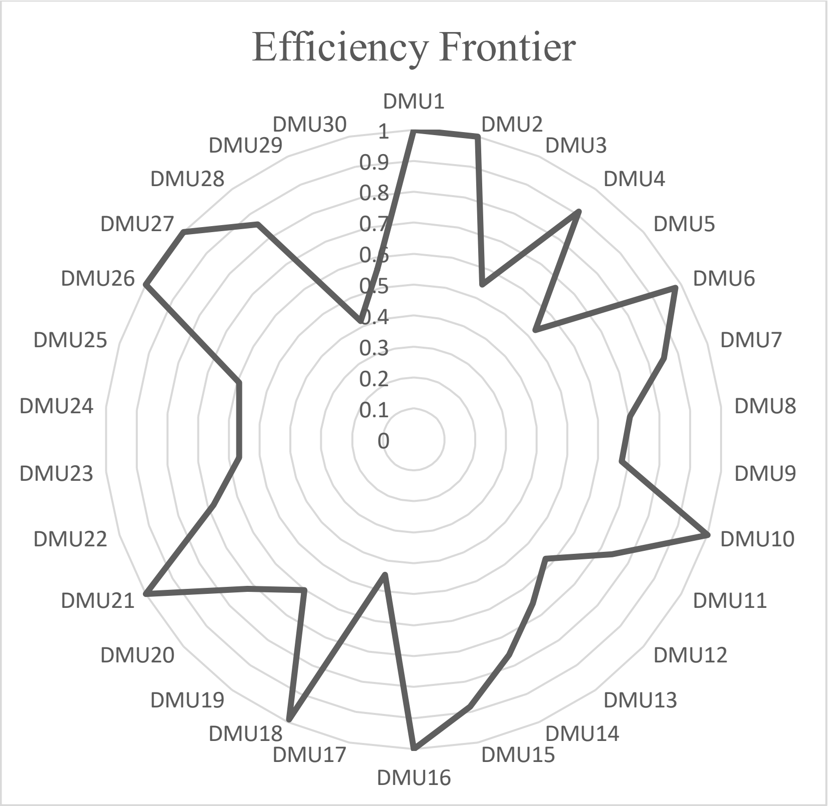

Figure 3. Technical efficiency of the 30 DMUs evaluated
Source: created by the authors

Table 7 presents a comparative analysis of the efficiency results obtained using DEA, establishing a comparative relation between the seven wells that were found to be on the efficiency frontier and their location in the hierarchy established through financial indicator analysis performed at PEMEX.

This comparison shows that of the seven allocations evaluated as efficient using DEA, only three are among the top three positions in PEMEX's financial indicator analysis, with the other four DMUs that are also on the efficiency frontier with DEA ranking 8th, 16th, 27th, and 28th.

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Table 7
Comparison of hierarchization
Comparative analysis of DEA vs. financial indicators

Allocation	DEA hierarchy	Allocation	PEMEX hierarchy
16DMU	1	1DMU	1
10DMU	1	2DMU	2
26DMU	1	10DMU	3
2DMU	1	4DMU	4
21DMU	1	6DMU	5
27DMU	1	7DMU	6
1DMU	1	11DMU	7
18DMU	0.98941693	21DMU	8
6DMU	0.97851623	12DMU	9
4DMU	0.90939874	9DMU	10
15DMU	0.88250099	14DMU	11
28DMU	0.85877482	15DMU	12
7DMU	0.85215713	DMU29	12
14DMU	0.76129046	8DMU	14
11DMU	0.74056402	5DMU	15
20DMU	0.72154223	16DMU	15
8DMU	0.70319000	18DMU	15
22DMU	0.68111247	17DMU	18
9DMU	0.67578981	23DMU	19
13DMU	0.65441908	13DMU	20
19DMU	0.60187758	3DMU	21
25DMU	0.59320128	20DMU	22
12DMU	0.5748543	25DMU	23
23DMU	0.56772876	19DMU	24
24DMU	0.56768248	30DMU	25
30DMU	0.56251646	22DMU	26
3DMU	0.54765841	27DMU	27
5DMU	0.52956473	26DMU	28
17DMU	0.4476646	24DMU	29
29DMU	0.41916741	28DMU	30

Source: created by the authors

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Conclusions

The main purpose of this study was to evaluate the relative efficiency of PEP oil allocations using DEA and to compare these results with those obtained using economic profitability indicators, in order to benchmark the oil allocations that comprise PEP's investment portfolio.

A comparative analysis verified that only three oil allocations, located on the efficiency frontier using DEA, are also among the top seven when evaluated using economic profitability indicators.

The comparative analysis also verified that two allocations considered efficient using DEA are in the last positions (27 and 28) when using the economic analysis performed at PEMEX, and that two of the allocations among the top seven positions in the economic analysis have efficiencies below 85%.

Using the VRS model for this analysis enables the identification of DMUs that operate in regions of economies of scale, diseconomies of scale, or constant returns to scale. This information is valuable for strategic decision-making, such as the expansion or contraction of operations.

By using DEA to determine the type of return to scale for each DMU analyzed, it will be possible to identify those DMUs with increasing returns to scale, to which greater budgetary resources can be allocated to increase their production.

In this case, using information from the slack variables and returns to scale, it will be possible to reallocate the budget from certain units that do not utilize it to those that operate with positive returns to scale, thereby improving their efficiency and production.

By determining the technical efficiency of each oil allocation analyzed, it has been possible to identify the underlying technology each oil well shares, which is rarely known.

Based on the technical efficiency results for each oil well, the underlying technology has been identified, i.e., the technological coefficients for which mathematical programming can be used to allocate budgets in the following PEP investment portfolios.

With the available information and the relative efficiency values for each oil allocation, the observed deviation of each from those considered most efficient has been determined, which will allow strategic planning to indicate the actions to be taken to improve the results.

By using DEA to perform comparative classification to form the PEP investment portfolio, considering the technical efficiency of the allocations, it is possible to have a reliable optimization tool to solve the budget allocation problem in PEP and, therefore, obtain a more robust investment portfolio.

Using the primal and dual structures outlined to solve the DEA model with linear programming, it is possible and extremely useful to interpret the results economically, as inefficient DMUs can be analyzed and opportunities to improve decision-making identified.

The information generated by the DEA is reliable and accurate for decision-making. It is also useful for proposing improvement policies for inefficient allocations based on the best policies for allocations on the efficiency frontier.

Therefore, it is recommended to implement and use DEA analysis to generate information and enhance decision-making certainty. This will provide an optimization tool to address budget allocation in PEP, using the best policies developed from oil allocations that maximize technical efficiency, thereby creating a more robust investment portfolio.

References

Alberto Jaime, J. (2016). Formulaciones en el análisis envolvente de datos (dea). resolución de casos prácticos. Sevilla: Universidad de Sevilla. Facultad de matemáticas.

Alidrisi, H. A. (2019). Monitoring the performance of petrochemical organizations in Saudi Arabia using data envelopment analysis. Mathematics, 7(6), 519. https://doi.org/10.3390/math7060519

Atris, A. M. (2020). Assessment of oil refinery performance: Application of data envelopment analysis-discriminant analysis. Resources Policy, 65, 101543. https://doi.org/10.1016/j.resourpol.2019.101543

Banker, R. D. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management science, 30(9)., 1078-1092. https://doi.org/10.1287/mnsc.30.9.1078

Barros, C., & Antunes, O. S. (2014). Productivity change in the oil blocks of Angola. Energy Sources, Part B: Economics, Planning, and Policy, 9(4), 413-424. https://doi.org/10.1080/15567249.2010.497794

Bezerra, P. R. C., Vieira, M. M., & de Almeida, M. R. (2017). Comparative Analysis About the Application of Methods of the Data Envelopment Analysis (DEA) in the Oil Industry. International Journal of Engineering Sciences & Research Technology. 6(3), marzo. https://doi.org/10.5281/zenodo.345699

Bogetoft, P., & Otto, L. (2010). Benchmarking with dea, sfa, and r (Vol. 157). Springer Science & Business Media. https://doi.org/10.1007/978-1-4419-7961-2_2

Centro de Investigación Económica y Presupuestaria, A.C. (CIEP). (2023). La importancia fiscal de PEMEX: hacia la era post petróleo. Ciudad de México: Centro de Investigación Económica y Presupuestaria A.C. Disponible en: https://ciep.mx/neQW. Consultado: 08/XI/2023

Charnes, A., & Cooper, W. W. (1962). Programming with linear fractional functionals. Naval Research logistics quarterly, 9(3-4), 181-186. Charnes, A. C. (1978). Measurement of

Productive Efficiency. European Journal of Operational Reseach. Serie A, Vol. 3, 429-444. https://doi.org/10.1002/nav.3800100123

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. European journal of operational research, 2(6), 429-444. https://doi.org/10.1016/0377-2217(78)90138-8

Chen, C. M., & Wang, H. (2024). Comparing eco-efficiency with productive efficiency: Addressing the dimensionality issue. European Journal of Operational Research, 313(3), 1170-1179. https://doi.org/ 10.1016/j.ejor.2023.09.001

Cooper, W. W., Seiford, L. M., & Zhu, J. (2011). Handbook on data envelopment analysis. Second Edition. New York: International Series in operations Research & management Science. Springer. https://doi.org/10.1007/978-1-4419-6151-8

Dalei, N. N., & Joshi, J. M. (2020). Estimating technical efficiency of petroleum refineries using DEA and tobit model: An India perspective. Computers & Chemical Engineering, 142, 10704.
https://doi.org/10.1016/j.compchemeng.2020.107047

Farrell, M. J. (1957). The measurement of productive efficiency. Journal of the Royal Statistical Society Series A: Statistics in Society, 120(3), 253-281. https://doi.org/10.2307/2343100

Hatami-Marbini, A., Arabmaldar, A., & Asu, J. O. (2022). Robust productivity growth and efficiency measurement with undesirable outputs: evidence from the oil industry. Or Spectrum, 44(4), 1213-1254. https://doi.org/10.1007/s00291-022-00683-y

Keynes, J. M. (2014). Teoría general de la ocupación, el interés y el dinero. Fondo de Cultura Económica. ISBN: 9786071624284

Koopmans, T. (1976). Concepts of optimality and their uses. The scandinavian Journal of Society 78(4), 542-560. https://doi.org/10.2307/3439525

López Bautista, G. (2020). Jerarquización de Asignaciones de PEMEX Exploración y Producción en términos de eficiencia, aplicando la ténica de análisis de Envolvente de Datos (DEA). [Tesis de maestría, Universidad Nacional Autónoma de México]. Repositorio institucional - Universidad Nacional Autónoma de México.

Ohene-Asare, K., Turkson, C., & Afful-Dadzie, A. (2017). Multinational operation, ownership and efficiency differences in the international oil industry. Energy Economics, 68, 303-312. https://doi.org/10.1016/j.eneco.2017.10.006

Oliveira, M. S. D., Lizot, M., Siqueira, H., Afonso, P., & Trojan, F. (2023). Efficiency analysis of oil refineries using DEA window analysis, cluster analysis, and Malmquist productivity index. Sustainability, 15(18), 13611. https://doi.org/10.3390/su151813611

PEMEX. (2018). Anuario Estadístico . Disponible en: https://www.pemex.com/ri/Publicaciones/Anuario Estadistico%20Archivos/anuario-estadistico_2018.pdf. Consultado: 23/10/23

PEMEX. (2022). Anuario Estadístico . Disponible en: https://www.pemex.com/ri/Publicaciones/Anuario%20Estadistico%20Archivos/Anuario%202022_VF.pdf. Consultado: 23/10/23

Sanchez-Robles, B., Herrador-Alcaide, T. C., & Hernández-Solís, M. (2022). Efficiency of European oil companies: an empirical analysis. Energy Efficiency, 15(8), 63. doi.org/ 10.1007/s12053-022-10069-2

Sodani, P. R. (2011). Ramanathan, R. An Introduction to data envelopment analysis: A tool for performance measurement. SAGE Publications, New Delhi, 2003, pp. 201. Rs 320. Journal of Health Management, 13(1), 113-114. https://doi.org/10.1177/097206341001300107

Tavana, M., Khalili-Damghani, K., Arteaga, F. J. S., & Hosseini, A. (2019). A fuzzy multi-objective multi-period network DEA model for efficiency measurement in oil refineries. Computers & Industrial Engineering, 135, 143-155. https://doi.org/10.1016/j.cie.2019.05.033

Vikas, R. V. (2019). Efficiency evaluation of Indian oil and gas sector: Data envelopment analysis. International Journal of Emerging Markets. 14(2), 362-378. https://doi.org/10.1108/ijoem-01-2018-0016

Wang, Y., Yao, L., Cui, S., & Zhu, Z. (2022). Eco-efficiency Assessment of Chinese Petrochemical Enterprises: A Data Envelopment Analysis Approach. Energy & Environment, 33(6), 1160-1180. https://doi.org/10.1177/0958305X211065424